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Research

Journal of Oil, Gas and Petrochemical Sciences

Derivation of the nonlinear dynamics of the interface between a kick gas fluid and mud system in a gas wellbore based on momentum conservation principle

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Received: November 10, 2018 | **Published:** January 01, 2019

Abstract

In this paper, the dynamics of the interface between a kick gas fluid and drill mud in the annular space of a drill hole has been considered using momentum balance approah. In so doing an existing momentum balance equation for mixture flow in conduits has been used to derive an interfacial velocity equation for aparticular scenario of kick event, where mud circulation ceases after kick detection. The model shows that the interface dynamics is non-linear. The utility of the derived equation stems from the fact that all parameters required for computation of velocities can be obtained. Based on the interfacial velocity model, the flux equation for annular mud flow and the maximum interfacial velocity at well head have been derived. The relevance of the velocity equation to casing design and running has also been dicussed. The approach in this work is purely analytical.

Keywords: momentum, velocity, kick, mud, expansion, interface acceleration

Introduction

Due to its strong bearing on the prospects of the oil and gas industry, notably from the point of view of drilling safety, drilling time and cost effective drilling operations, the kick problem has received a great deal of attention from the industry and academia.[1–4] Consequently, modeling the kick phenomenon as a means of thoroughly understanding its occurrence and seeking mitigating strategies has been carried out.[2,4–6]

Regarding the modeling task, much work has been done in connection with annular pressure evolution during kick event and control.[7–9] Also, gas migration velocity during a gas kick event has been given due attention.[3] In the work of Nunes et al.,[10] other notable contributions to our understanding of kick have been cited.[2,11–14] It is the view of the present paper that while the annular pressure aspect of gas kick has received sufficient attention, not much has been done regarding the velocity or interfdacial dynamic aspects. This is because, in the work of Johnson and Cooper,[3] the major attention was on the rate of gas migration in the annular space. While a thorough understanding of kick and its potential impact on the drilling industry is worth, knowledge of non-linear systems that are characterized by expansion motions of compressiblefluids as encountered in the nozzles of rockets engines gives a clue that the interface of a kick gas-mud system can equally be described non-linearly, due to increasing expansion of the gas phase in the annular space.[15] Therefore, the objective of this paper is to consider the mathematical modeling of the interface dynamics. Specifically, the motion of the upper interface of a gas kick fluid in a wellbore during a kick event will be considered to derive both accelerationand velocity equations. The accomplishment of this task will augment the work of Johnson and Cooper,[3] in addition to providing the impetus for related simulation works where the need arises.

Present Model

The first mathematical modelling of a gas kick was proposed in 1968. This model disregarded friction pressure loses in the annular space, slippage velocity between mud and gas as well as gas solubility in mud.[16] In this paper, it isfirst assumed that the kick fluid enters the well bore due to underbalanced drilling conditions and expands as it moves through the annular column while annular circulation continues. Later, this assumption will be modified to suit the objective. Based on the assumption, the following hold true:

1. The upper boundary of the gas kick fluid will accelerate as it moves through the annular space due to expansion.
2. The velocity of the upper interface will be greater than that of annular mud circulating velocity and this will cause an increasing gas column.
3. The upper interface of the system will experience a time varying acceleration.
4. As the kick fluid moves through the annular space, bottom hole pressure will decrease due to decrease in annular mud column.
5. Maximum bottom hole pressure will be experienced immediately the kick fluid enters the well bore and the least bottom whole pressure will be experienced when the kick fluid reaches well head.

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There are different types of kick gas influxes into the annular space. One is where the influx occurs as a dispersion of bubbles in the annular space (See Figure 1). The second case involves influxes of discontinuous slugs of kick gas in the annular space. Third type occurs as a distinct slug with a sharp interface with annular drill mud (See Figure 2). The following assumptions will be made to simplify the modeling approach:

1. The kick fluid enters the wellbore as a slug.
2. A water base mud is assumed, meaning the solubility of gas is reasonably negligible.
3. The density of the water base mud is assumed to be constant.
4. Momentum conservation law holds for the motion of the drill mud above the upper interface of the gas kick fluid-water base mud system.
5. Changes in mud density with distance is negligible and change in mud temperature with depth is also negligible. Thus:

$$\frac{d\rho_m}{dZ} = 0$$

6. The interface between gas and mud is distinct and vertical to wellbore axis and changes in drill pipe outer diameter and hole size due to hole washout is negligible.[17]
7. Phase transition due to vaporization is also negligible.[17]

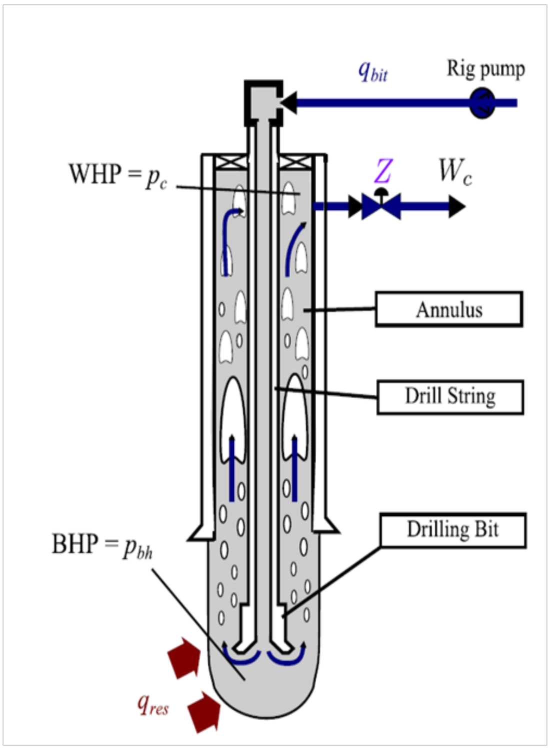

Figure 1 Schematics of a kick gas influx as a mixture in annular drill mud.[6]

In attempting the mathematical model of the interfacial dynamics of a gas kick in this paper, the motivation stems from the fact that the motion of the system, gas kick fluid-mud obeys the fundamental laws of fluid dynamics, notably those of momentum and mass.[18]

Accordingly, the appropriate momentum conservation law will be invoked for achieving the objective. Consequently, modification of the momentum equation for flow in an annular space for the sake of the present work gives[19]:

$$-\frac{\partial P}{\partial z} = \left[\phi\rho_1 + (1-\phi)\rho_2\right]g \pm \frac{R_e}{2d_h}\left[\phi\rho_1 V_1^2 + (1-\phi)\rho_2 V_2^2\right] + \left[\phi\rho_1\frac{dV_1}{dt} + (1-\phi)\rho_2\frac{dV_2}{dt}\right] \quad (1)$$

Where:

P = pressure in then flowing fluid

ρ_1 = density of component one

ρ_2 = density of component 2

φ = volume fraction of component 1

g = acceleration due to gravity

R_e = Reynolds number for mixture flow

d_h = hydraulic diameter of conduit

V_1 = velocity of phase 1

V_2 = velocity of phase 2

z = distance

t = time

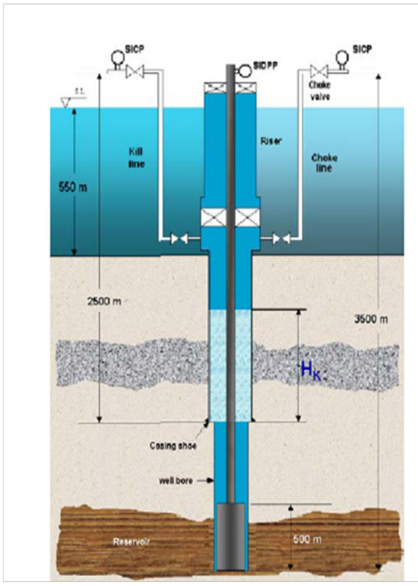

Figure 2 Schematics of a kick gas influx as a slug.[10] H_K = thickness of gas kick fluid.

If the kick gas enters the well bore as a slug, and exists distinctly from the drill mud, the flow of mud above the upper interface of the kick gas can be described by a modified form of Eq. (1) as:

$$-\frac{\partial P}{\partial z} = \left[\phi\rho_1\right]g \pm \frac{R_e}{2d_h}\left[\phi\rho_1 V_1^2\right] + \left[\phi\rho_1\frac{dV_1}{dt}\right] \quad (2)$$

Here, φ = 1. Thus, for upward flow of drill mud the following equation holds:

$$-\frac{\partial P}{\partial z} = \rho_m g + \frac{R_e}{2d_h}\rho_m V_m^2 - \rho_m\frac{dV_m}{dt} \quad (3)$$

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V_m = mud velocity

ρ_m = mud density

The distance travelled by the upper interface of the kick fluid-drill mud system per unit time, which is the velocity (V_1), for flow in the upward direction is:

$$\frac{dZ(t)}{dt} = V_1$$

$$\frac{d^2Z(t)}{dt^2} = \frac{dV}{dt} \qquad (4)$$

In which $Z(t)$ is the distance travelled upward at a given time, t. Hence:

$$-\frac{\partial P}{\partial z} = \rho_m g + \frac{R_e}{2d_h}\rho_m\left(\frac{dZ(t)}{dt}\right)^2 + \rho_m\frac{d^2Z(t)}{dt^2} \qquad (5)$$

Equation (5) can be written in integral form as:

$$-\int_{P_{sur}}^{P(Z)} dP = \int_{Z(t)}^{D}\rho_m g dZ + \int_{Z(t)}^{D}\frac{R_e}{2d_h}\rho_m\left(\frac{dZ(t)}{dt}\right)^2 dZ - \int_{Z(t)}^{D}\rho_m\frac{d^2Z(t)}{dt^2}dZ \qquad (6)$$

Hence:

$$P(Z(t)) - P_{sur} = \rho_m g(D - Z(t)) + \frac{R_e}{2d_h}\rho_m\left(\frac{dZ(t)}{dt}\right)^2(D - Z(t)) - \rho_m\frac{d^2Z(t)}{dt^2}(D - Z(t)) \qquad (7)$$

Where:

$P(Z(t))$ = is the pressure in drill mud above the upper interface of drill mud at a given distance, which is a function of time.

Pressure balance

Considering Figure 3, it is assume that the initial column of the kick fluid is Z_0, and that annular mud velocity is V_{an}.

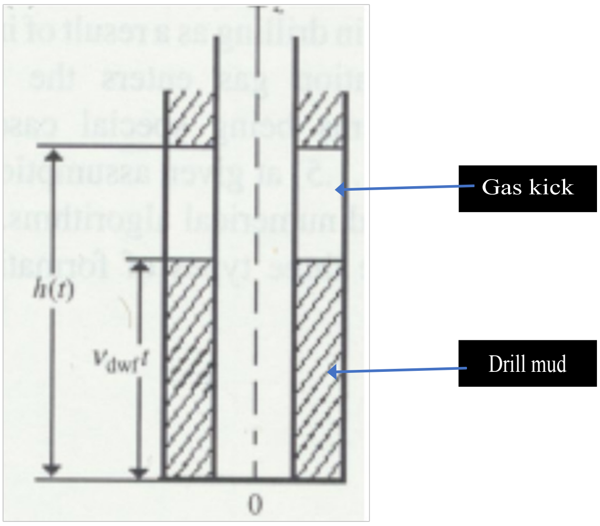

Figure 3 Schematics of a gas kick influx in a well bore.

The initial bottom hole pressure at the time of the kick event is given as:

$$(D - Z_0)\rho_m g + P_{suf} = P_b \qquad (8)$$

Where:

D = depth of well at the time of the kick event

Z_o = initial column thickness of kick fluid

P_{ib} = initial bottom hole pressure

P_{suf} = surface pressure

If the pressure at the upper interface at a given time, t, is the same as that averaged over the column of kick fluid, the application of the state equation for the gas kick gives:

$$P(Z)\left[Z - V_{an}t\right]A_n / T_z Z_Z = \left[P_{sur} + (D - Z_0)\rho_m g\right]Z_0 A_{an} / T_{Z_0} Z_{Z_0} \qquad (9)$$

Where:

T_Z = temperature at annular position Z

Z_Z = gas compressibility factor at annular position Z

T_{Z_0} = temperature at annular position Z_0

Z_{Z_0} = gas compressibility factor at annular position Z

Solution for pressure at a given distance using Eq. (9) gives:

$$P(Z) = \beta\left[P_{sur} + (D - Z_0)\rho_m g\right]Z_0 / \left[Z(t) - V_a t\right] \qquad (10a)$$

Where:

$$\beta = \frac{T_Z Z_Z}{T_{Z_0} Z_{Z_0}} \qquad (10b)$$

Substituting into Eq. (7) gives:

$$\beta\frac{\left[P_{sur} + (D - Z_0)\rho_m g\right]Z_0}{(Z(t) - V_{an}t)} - P_{sur} = \rho_m g(D - Z(t)) + \frac{R_e}{2d_h}\rho_m\left(\frac{dZ(t)}{dt}\right)^2(D - Z(t))$$

$$+\rho_m\frac{d^2Z(t)}{dt^2}(D - Z(t)) \qquad (11)$$

Thus:

$$\beta\frac{\left[P_{sur} + (D - Z_0)\rho_m g\right]Z_0}{(Z(t) - V_{an}t)(D - Z(t))} - \frac{P_{sur}}{(D - Z(t))} - \rho_m g = \frac{R_e}{2d_h}\rho_m\left(\frac{dZ(t)}{dt}\right)^2 + \rho_m\frac{d^2Z(t)}{dt^2} \qquad (12)$$

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Equation 13 is a second order ordinary differential equation of second degree and it is nonlinear in character. The solution for velocity V_m, can be obtained using the derivative substitution method.[20] In this regard, the following substitution will be made:

$$\frac{dZ(t)}{dt} = P \tag{13}$$

$$\frac{d^2Z(t)}{dt^2} = \frac{dP}{dt} = \frac{1}{2}\frac{d(P^2)}{dZ(t)}\frac{dZ(t)}{dt} = \frac{1}{2}\frac{d(P^2)}{dZ(t)}$$

Substituting and dividing through by density gives:

$$2\beta\frac{\left[P_{sur}+(D-Z_0)\rho_m g\right]Z_0}{\rho_m\left(Z(t)-V_{an}t\right)\left(D-Z(t)\right)} - \frac{2P_{sur}}{\rho_m\left(D-Z(t)\right)} - 2g = \frac{R_e}{d_h}P^2 + \frac{d(P^2)}{dZ(t)} \tag{14}$$

The equation is now a first order linear homogenous ordinary differential equation.

The integration factor is:

$$I = e^{\int \frac{R_e}{d_h}dZ} = e^{\frac{R_e}{d_h}Z(t)} \tag{15}$$

Multiplying through by this factor gives:

$$2\beta\frac{\left[P_{sur}+(D-Z_0)\rho_m g\right]Z_0}{\rho_m\left(Z(t)-V_{an}t\right)\left(D-Z(t)\right)}e^{-\frac{R_e}{d_h}Z(t)} - \frac{2P_{sur}}{\rho_m\left(D-Z(t)\right)}e^{-\frac{R_e}{d_h}Z(t)} - 2ge^{-\frac{R_e}{d_h}Z(t)} = \frac{R_e}{d_h}e^{-\frac{R_e}{d}Z(t)}P^2$$

$$+e^{-\frac{R_e}{d_h}Z(t)}\frac{d(P^2)}{dZ(t)} \tag{16}$$

The following parameters will be defined:

$$\frac{R_e}{d_h} = \xi$$

$$2\beta\frac{\left[P_{sur}+(D-Z_0)\rho_m g\right]Z_0}{\rho_m} = \zeta \tag{17}$$

$$2\frac{P_{sur}}{\rho_m} = \lambda$$

Substituting into Eq. (16) gives:

$$\frac{\zeta}{\left(Z(t)-V_{an}t\right)\left(D-Z(t)\right)}e^{-\xi Z(t)} - \frac{\lambda}{\left(D-Z(t)\right)}e^{-\xi Z(t)} - 2ge^{-\xi Z(t)} = \xi e^{-\xi Z(t)}P^2$$

$$+e^{-\xi Z(t)}\frac{d(P^2)}{dZ(t)} \tag{18}$$

Equation (18) can be written as:

$$\frac{d}{dZ(t)}\left(e^{\xi Z}P^2\right) = \frac{\zeta}{\left(Z(t)-V_{an}t\right)\left(D-Z(t)\right)}e^{\xi Z} - \frac{\lambda}{\left(D-Z(t)\right)}e^{\xi Z} - 2ge^{\xi Z(t)} \tag{19}$$

Equation (19) can be solved for two distinct scenarios in drilling, where a gas kick is encountered at a given depth. One is the case where mud circulation is quickly stopped following pit gain. In this case, the annular mud velocity in Eq. (19) becomes zero. The former scenario will be the focus of this paper, assuming the surface pressure, P_{sur}, is atmospheric.

Solution for the case with aero mud velocity

In drilling practice, the following are recommended when a kick is taken[21]**:**

i. Shut in the well

ii. Apply back pressure through the choke while continuing

The essence of the above steps is to detect continuous pit gain through annular mud flow, which is the criterion for confirming the existence of a kick. Hence, Eq. (19), which is the result of the derivative substitution method can be solved for the square of mud velocity after shut-in, which is indicative of the flow of a kick fluid in the well bore following the initial influx. Figure 4 shows the schematics of this scenario.

Thus:

$$\frac{d}{dZ(t)}\left(e^{\xi Z}P^2\right) = \frac{\zeta}{Z(t)\left(D-Z(t)\right)}e^{\xi Z} - \frac{\lambda}{\left(D-Z(t)\right)}e^{\xi Z} - 2ge^{\xi Z} \tag{20}$$

Integration gives:

$$P^2 = \frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln(D-Z) - \frac{2g}{\xi} + C \tag{21}$$

Substitution of P from Eq. (13) gives:

$$\left(\frac{dZ(t)}{dt}\right)^2 = V_{\text{int}}^2 = \frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln(D-Z) - \frac{2g}{\xi} + C \tag{22}$$

V_{int} = velocity of the interface between kick fluid and drill mud

Solution for the square of interface velocity will assume the following boundary condition:

$$Z = Z_0, \quad V_{\text{int}} = 0 \tag{23}$$

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Substituting these boundary conditions and solving for the integration constant, C, gives the interface velocity as:

$$V_{int}^2 = \frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln(D-Z) - \frac{2g}{\xi} - \frac{\zeta}{D}\ln\left(\frac{Z_0}{D-Z_0}\right) - \lambda\ln\left(D-Z_0\right) + \frac{2g}{\xi} \quad (24a)$$

This implies:

$$V_{int}^2 = \frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln(D-Z) - \frac{\zeta}{D}\ln\left(\frac{Z_0}{D-Z_0}\right) - \lambda\ln\left(D-Z_0\right) \quad (24b)$$

Setting $-\frac{\zeta}{D}\ln\left(\frac{Z_0}{D-Z_0}\right) - \lambda\ln\left(D-Z_0\right) = \eta$,

Eq. (24b) becomes:

$$V_{int}^2 = \frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln(D-Z) + \eta \quad (25)$$

The final velocity equation gives:

$$V_{int} = \left[\left\{\frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln\left(D-Z\right) + \eta\right\}\right]^{1/2} \quad (26)$$

The series expansion is truncated at the third term assuming the depth of the well is too big to render subsequent terms negligible. Thus:

$$V_{int} = \sqrt{\zeta\ln\left(\frac{Z}{D}\right) + \lambda\ln\left(D\right) - \eta - \left(\frac{\zeta}{D^2} - \frac{\lambda}{D}\right)Z - \left(\frac{\lambda}{2D^2}\right)Z^2} \quad (27)$$

Velocity isthe time derivative of distance as:

$$V_{int} = \frac{dZ}{dt} = \sqrt{\zeta\ln\left(\frac{Z}{D}\right) + \lambda\ln\left(D\right) - \eta - \left(\frac{\zeta}{D^2} - \frac{\lambda}{D}\right)Z - \left(\frac{\lambda}{2D^2}\right)Z^2} \quad (28)$$

This links time to distance as:

$$t = \int_{Z=Z_0}^{Z}\left[\sqrt{\zeta\ln\left(\frac{Z}{D}\right) + \lambda\ln\left(D\right) - \eta - \left(\frac{\zeta}{D^2} - \frac{\lambda}{D}\right)Z - \frac{\lambda}{2D}}\right]^{-1} dZ \quad (29)$$

Interface velocity at wellhead

The actual distance travelled by the upper interface of the mud-kick gas system is given as;

$$Z^* = Z - Z_0 \quad (30)$$

Where:

Z^* = actual distance travelled by the upper interface

Z_0 = initial kick gas thickness in the annular space

Thus:

$$\frac{dZ^*}{dt} = \frac{dZ}{dt} - \frac{dZ_0}{dt} = \frac{dZ}{dt} \quad (31)$$

This means when the upper interface gets to the well head, the distance travelled will be:

$$Z^* = D - Z_0 \quad (32)$$

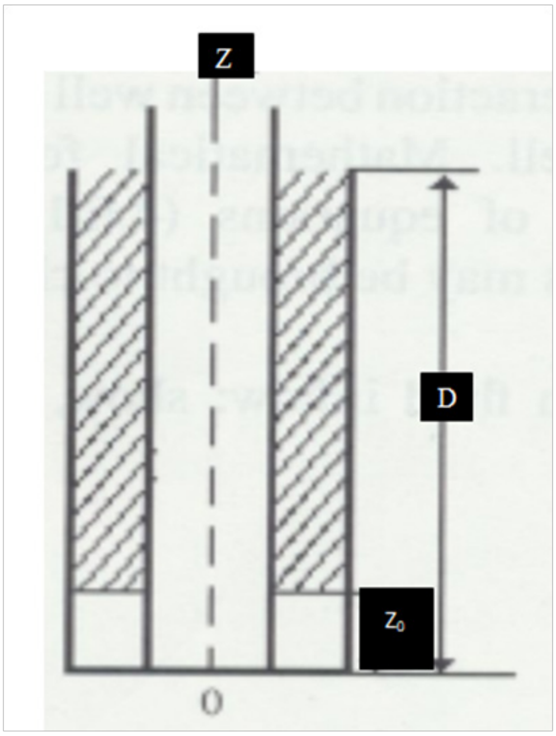

Figure 4 schematics of a gas kick with no annular circulation.

Equation (26) can be interpreted from two possible scenarios, following the sudden influx of a kick fluid into the well bore. Thus, for a very deep well with a very thick initial mud column above the kick fluid with an initial column thickness Z_0, the gas kick will expand against a significant load of mud column and eventually come to rest. During this time period of expansion, there will be a continuous pit gain following the initial pit gain. When the interface eventually comes to rest in the annular column, additional pit gain will cease. The interface velocity will be zero. Equating Eq. (26) to 0 gives:

$$0 = \left[\left\{\frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln\left(D-Z\right) + \eta\right\}\right]^{1/2} \quad (33)$$

If Z_f is the solution to Eq. (33), the additional distance travelled by the interface is given:

$$Z_f - Z_0$$

However, for a shallow well where the volume of influx is quite significant with a small thicknessof drill mud column above the initial interface, expansion of the kick fluid results in the interface reaching well head. Under this condition, the solution to Eq. (26) gives the velocity of the interface at well head and permitssetting $Z = D - Z_0$. Thus:

$$V_D = \left[\left\{\frac{\zeta}{D}\ln\left(\frac{D-Z_0}{Z_0}\right) + \lambda\ln\left(D-D+Z_0\right) + \eta\right\}\right]^{1/2} = \left[\frac{\zeta}{D}\ln\left(\frac{D}{Z_0} - 1\right) + \lambda\ln\left(Z_0\right) + \eta\right]^{1/2} \quad (34)$$

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Where:

V_D = interface velocity at well head

The situation described by Eq.(34) leads to a complete mud loos in the hole, which can lead to eminent blowout.

Flux of mud in annular space

The existence of interface velocity means there will be a definite mud flux in the annular space defined as the mass flow rate of drill mud per unit annular cross sectional area. At this point, it is appropriate to divide the motion of drill mud above the upper interface of the mud-gas kick system into two with different motion characteristics. In this regard, it is intuitive to accept that as the kick fluid begins to expand, the column of mud above the upper interface will have two distinct velocity zones, such that close to the interface, the volume of mud will have velocity equal to that of the interface. Also, far away from the interface, the velocity will be different. Tarvin[22] equated gas rising velocity through mud to the product of the velocity of mud just above the gas and a factor plus the slip velocity of the gas as:

$$V_g = KV_m + V_s \tag{35}$$

Where:

V_g = velocity of rising gas

K = factor

V_m = velocity of mud just above the gas

V_s = slip velocity of gas

For industrial standard K = 1. Thus:

$$V_g = V_m + V_s \tag{36}$$

In the present paper, the distinct zone of the gas kick fluid means the slip velocity is not applicable and this amounts to saying the velocity of mud just above the interface is equal to the velocity of the interface. This equation supports the assumption of the present paper that the velocity of mud just above the interface is equal to the interface velocity. Consequently, the mass flux of drill mud close to the interface defined as the product of interface velocity and density of drill mud can be written as:

$$j_m = \rho_m V = \rho_m \left[\left\{\frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln\left(D-Z\right) + \eta\right\}\right]^{1/2} \tag{37}$$

Where:

j_m = mass flux [kgm^{-2} s^{-1}]

The mass of mud per unit time in the annular space is, therefore,

$$\dot{m} = 0.25\pi d_h^2 \rho_m V = \left[\left\{\frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln\left(D-Z\right) + \eta\right\}\right]^{1/2} 0.25\rho_m \pi d_h^2 \tag{38}$$

Where:

j_m = kgs^{-1}

$\dot{m}$ = mass flow rate

Accordingly, the dynamic pressure, which is the pressure possessed by a flowing fluid is given as:

$$P_{dy} = \frac{1}{2}\rho_{gkg} V_{\text{int}}^2 = \rho_{kg}\left[\left\{\frac{\zeta}{D}\ln\left(\frac{Z}{D-Z}\right) + \lambda\ln\left(D-Z\right) + \eta\right\}\right] \tag{39}$$

Where:

ρ_{kg} = density of kick gas

Determination of initial annular column thickness of gas kick

Equation (28) and Eq. (32) contain information about the initial annular column thickness of kick gas, Z_0. Information about this parameter can be obtained from a volume balance equation involving the volume of gas column and the pit gain during the initial influx. Thus:

$$Z_0 = \frac{G}{A_{an}} = \frac{G}{0.25\pi d_h^2} \tag{40}$$

Where:

G = pit gain

d_h = hydraulic diameter of annular space

With this information, the parameters η and ς can be computed for a given pit gain in barrels and annular hydraulic parameter.

Maximum interface velocity corresponding tomMinimum bottom hole pressure

For all cases considered with or without annular mud circulation, the maximum bottom hole pressure is exerted at the instance of kick gas influx into the annular space. As the kick gas expands in the annular column, annular gas column increases while annular mud column decreases. This causes a gradual decrease in bottom hole pressure until it becomes a minimum when the upper interface of the gas-kick fluid-mud system reaches well head. The following sections will be devoted to the applicability of the interfacial velocity model and discussion.

Applicability of model

Application of the model for interfacial velocity calculation requires the following steps and data:

Step 1: Obtain information about gas kick volume from pit gain

Step 2: Obtain information about hydraulic diameter of hole (d_h), by calculating equivalent diameter based on outer diameter of drill pipe assembly and drill collar assembly using equation in.[23]

Step 3: Obtain information about surface pressure

Step 4: Obtain information about mud density and mud dynamic viscosity

Step 5: Using relevant data, calculate parameters defined by Eq. (14)

Step 6: Calculate interfacial velocity using the appropriate model

Discussion

Relationship of interfacial velocity to expansion of exhaust gases in rocket nozzle

The interface velocity equation in this paper (Eq. (26)) has been derived for the case of an expanding gas in the annular space of a

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drill hole. This means the interface velocity increases with expansion in the annular column. A dynamic and widely applicable technology related to the expansion of combustion gases is that related to rocket science. In rocket launching technology, a propellant consisting of a mixture of combustion supporting gas (oxygen) and fuel (hydrogen or hydrocarbon gas) is burnt to release a combustion gas that expands through a nozzle (See Figure 5). Figure 5, based on rocket science, is another case of the velocity of gas increasing through expansion like that encountered in the motion of the gas kick fluid-mud system.

It is possible to calculate the velocity of the expanding gases at the exit. This velocity is called the exit velocity and it is given as[15]:

$$V_e = \sqrt{\frac{2\gamma}{\gamma-1}\frac{R_u}{M}T_c\left(1-\left(\frac{P_e}{P_c}\right)^{(1-\gamma)/\gamma}\right)} \tag{41}$$

Where:

V_e = exit velocity

R_u = universal gas constant

P_e = exit pressure

T_c = chamber temperature

P_c = chamber pressure

M = molecular mass of gas:

γ = ratio of apecific heat capacities

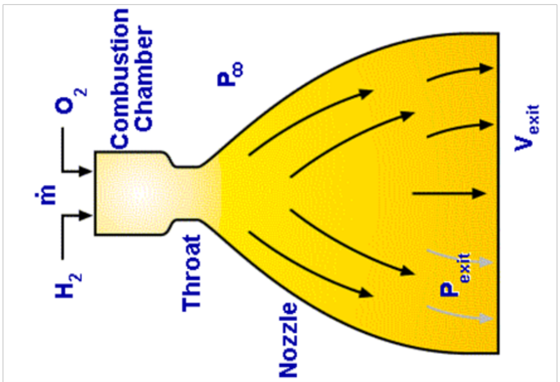

Figure 5 Expansion of combustion gases through a rocket nozzle.

The velocity equation at the well head obtained in the present paper is recalled as:

$$V_D = \left[\frac{\zeta}{D}\ln\left(\frac{D}{Z_0}-1\right)+\lambda\ln\left(Z_0\right)+\eta\right]^{1/2}$$

The parameters are defined as:

$$2\beta\frac{\left[P_{sur}+\left(D-Z_0\right)\rho_m g\right]Z_0}{\rho_m}=\zeta$$

(17)

$$2\frac{P_{sur}}{\rho_m}=\lambda$$

Comparison of Eq. (41) to the Eq. (41)and realizing the definitions of the constants in Eq. (17) shows that they all contain the following:

1. Square root sign
2. Initial parameters

All parameters that appear in Eq. (41) are not variables.The initial and final parametersin Eq. (42) are chamber pressure, exit pressure and exit velocity respectively. In the case of the velocity equations in this paper, the initial and final parameters are the initial pressure $\left(D-Z_0\right)\rho_m g + P_{suf} = P_b$) and final or surface pressure P_{suf} . All other parameters are also constant.

Relationship of interfacial velocity to expansion of a rising bubble through a liquid

The expansion of a rising gas bubble through a liquid is a common occurrence related to the opening of a pressurized fluid in a container, such as beer, wine etc. Brennen[24] studied the interface growth rate of a bubble in a liquid. By using a pressure balance approach, he arrived at the following asymptotic growth rate of the interface:

$$\frac{dR}{dt}=\left\{\frac{2\left(PV-P_\infty^*\right)}{3\rho_L}\right\}^{1/2} \tag{42}$$

Where:

R = radius of bubble

P = pressure in bubble

V = volume of bubble

P_∞^* = pressure at an infinite distance from the interface

ρ_L = density of liquid

In Eq. (52), the velocity of the interface between the expanding bubble and the surrounding liquid is expressed in terms of the rate of increase of bubble radius, which is similar to the distance Z, in Eq. (26). Equation (52) contains a square root sign similar to Eq. (26) in the present paper.

The approach to the derivation if interfacial kick gas velocity is based on a gas slug model. InSoceity of Petroleum Engineers Texbook Series which are standards, the slug flow model has been used for the calculation of kick fluid density and the density of new mud required to drill after the kick fluid has been circulated out.[23] Therefore, the theoretical basis of the present paper has industrial significance.

In the literature, different models of gas kick in well bores have been developed in addition to those already cited at the beginning of this paper. Recently, He et al.[25] recognized the momentum conservation approach in addition to the mass conservation approach. In the present paper, the momentum conservation method has been central to model development. According to Marshall and Bentsen[26] when the depth of the well is less than 1500 meters, the temperature difference of the fluid in the wellbore is less than 25 K. Therefore, to begin a meaningful discussion, the interface velocity equation deserves to be considered. First, to identify various parameters that control it, and second, to understand how they actually control it for the case of a well depth below 1500 meters. In this regard, the effect of the parameters, ξ , ζ and λ can be discussed as follows:

The ratio of Reynolds number to hydraulic diameter finally disappears from the interfacial velocity equation while the other two parameters ζ and λ have effects. This means that increasing surface pressure while decreasing mud weight, which is related λ will cause interfacial velocity to increase. The fact that decreasing

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mud weight results in interfacial velocity increase predicted by the present model is also supported by the gas drift velocity (Eq.3) of Guo et al.[27] Likewise, decreasing surface pressure while increasing mud weight will negatively impact interfacial velocity. A positive effect of the parameter ζ ,can be realized by decreasing mud weight. Also, the definition of this parameter shows that the depth at which a kick is taken can impact interfacial velocity. In this regard, inspection of Eq. (24a) shows that when a gas kick occurs at a deeper depth, higher interfacial velocities will be experienced if the initial thickness of the kick fluid is smaller. The reason is that deeper depths afford enough time for expansion of the gas kick fluid leading to higher interfacial velocities.

In the study of dynamics, a clear distinction is always drawn btween linear and non-linear dynamics.[28] In the former, a body experiences a constant acceleration while in the latter, the acceleration is time variant.The case of a gas kick fluid-mud interface system is a typical example of the non-linear case. In this regard, if distance is substituted by the product of time and velocity and acceleration is substituted for the second derivate of distance with respect of time, it becomes easy to see from Eq.(12) that the acceleration is a nonlinear fuction of velocity, which is in turn a fuction of time as:

acceleration $\frac{d^2Z}{dt^2} = a\left(V\left(t\right)\right)$.

The interfacial velocity equation also shows that at a given time, the acceleration is also dependent on thermophysical and hydraulic parameters, these being mud density (thermphysical), surface pressure and the ratio of Reynolds number to hydraulic diameter, the latter being a hydraulic parameter.

Equation 26 predicts lower interfacial velocities for deeper depths of kick. The reason is that for deep eper wells, the difference bewtween well depth and interfacial position in the annumral space is bigger. This means for two wells with the same mud weight, surface pressure and wellbore assembly, the interface in a shallower well with the same initial kick fluid volume will havehiger interfacial velocity compared to that of adeeper well.

Equation (34) shows that the magnitude of the interfacial velocity at well head is governed by the ratioof kick depth to the initial gas kick thickness in the annular space given as D/Z_0 . The bigger this value, the higher the interfacial velocity at well head. The reason is that smaller values of gas thickness provide the opportunity for more expansion before getting to the well head, resulting in higher velocities. Generally the interfacial velocity will be a maximum at well head because of the depth factor in Eq.(34). This is because the thickness of the kick gas fluid compared to well depth is small. This makes the quatitity in the aurgument of the first logarithm term big. The fact that the interface velocity at well head will be maximum makes the case comparable to that of an exhaust gas from a rocket cumbustion chamber expanding and ecceleation through a divergent nozzle and attaining a maximum exit velocity at the exit.

Time for interface to travel to a given point

When a gas kick is taken during drilling, two notable points in the drill hole are of interest in light of formation fracturing and the occurrence of partial annular mud volume, each of which can cause the kick to degenerate into a blowout. Thenearest point of interest in the annular space from bottom hole is the formation below the last casing shoe. Figure 6 shows plots of pore pressure and fracture pressure versus depth. Accordingly, the figure shows that lower fracture pressures are encountered at shallower depths.

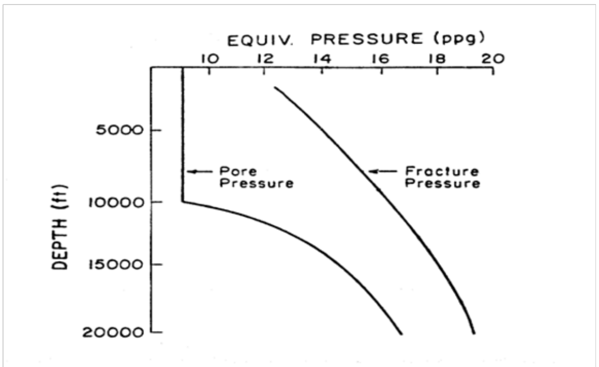

Figure 6 Plots of pore pressure and fracture pressure versus formation depth.

Therefore, the formation below the last casing shoe has the lowest fracture pressure in the next drilling interval. The implication is that for high pressure gas expanding through the annular space and passing the last casing shoe, (See Figure 7), the hydrostatic pressure above the interface plus the pressure in the gas slug cancan exceed the fracture pressure to cause formation fracturing in the vicinity.

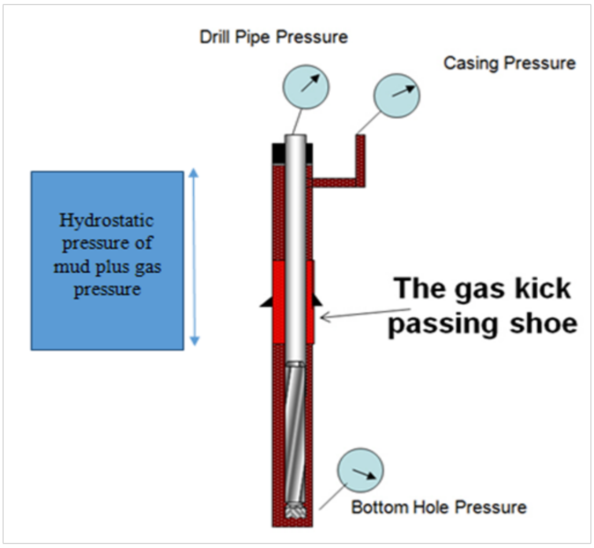

Figure 7 Schematics of kick gas rise past last casing shoe

ZAn idea about the time required for the interface to get to the last casing shoe can be obtained by numerically integrating Eq. (24). However, without completing the integration, it is possible to predict the effect of various parameters that appear in the velocity equation. This time is governed by some parameters. They are the following ξ, λ and η . The first two parameters are inversely proportional to mud weight. Therefore, increasing mud weight will decrease the time required for the interface to rise to the last casing shoe environment for a given kick depth and surface pressure. The effect of the third parameter on migration time can be seen by making the following assumptions:

At depth typical of gas kicks, the thickness of the kick fluid is very small compared with formation depth. This assumption permits the parameter to be simplified as:

$$\eta = -\frac{\zeta}{D}\ln\left(\frac{Z_0}{D}\right) - \lambda\ln\left(D\right)$$

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This equation links the importance of the ratio of initial kick fluid thickness to well depth to migration time. Accordingly, it shows that kicks with very small volumes will take longer times to migrate to the last casing shoe environment compared to kick gas fluids of bigger thicknesses.

Relevance of velocity equation to the worst case scenarios of casing running

The case of interfacce velocity at well head representing the lowest bottom hole pressure presented in this paper is directly applicable to casing design issues. Generally, one of the worst case scenarios encountered in casing design is the one related to a complete mud loss, where bottom hole pressure atagiven depth becomes atmospheric**.** In the practice of casing running, this can become possible because of excessive surge pressures that develop in response to annular mud flow along casing pipes following casing pipebody displacement of annular mud.[29] The dynamic friction pressure drop, when pipe lowering velocity exceeds a maximum value coupled with the hydrostatic pressure due to mud column at a depth can be high enough to exceed the fracture pressure at that depth, leading to fracturing which can cause whole mud loss. Therefore, under such conditions, the take home message of this paper is that if surge pressures were high enough to induce fracturing near bottom hole, then initial gas kick fluids with smaller volumes will take longer times to reach well head compared to those with bigger volumes. Accordingly, since higher pressures are encountered at deeper depths, the likelihood of biggerkick gas volumes being encountered at deeper depths causing substantial pit gains is higher. The modeling in this paper also shows the importance of higher surface pressures in suppressing gas kick fluid annular migration, where higher surface pressure translate to lower velocities and viceversa.

Generally, the behavior of a nonlinear system is described by a nonlinear system of equations. An alternative approach involves a description where the unknown function appears as a variable of another polynomialdegree higher than one. In this paper, we have shown mathematically, using momentum balance approach that the acceleration of a kick gas-mud interface in the annular space of a well bore can be described by a non linear combination of velocity and distance. Nonlinear dynamical systems that describe changes in variables with time can become chaotic, upredictable, or counterintuitive, being distinct from linear systems and that is why an insight into the nonlinear aspect of kick fluid dynamics is critical to efficient well control during drilling operations. At least, the non linearity of the interface dynamics revealed by this paper is a reminder that, to avoid chaos or unpredictability setting in during kick events, minimum response time for initiating effective well control measures is required on thepart of the drilling crew. This requirement can only be met where experienced hands are on the job.

In this paper, the established principle of momentum conservation in fluid flow has been used to derive the interface velocity of an expanding gas kick fluid in the annular space of a gas wellbore. The practical aspect of the derived equation has been discussed. What is more, the similarity of the derived equation to those of systems with non linear behavior has been pointed out.The following sum up the conclusion of this paper.

Conclusion

1. The motion of the interface of a gas kick-mud system is non-linear
2. The interface accelerates and achieves a maximum value when it gets to well head, where the pressure becomes atmospheric.
3. Lower mud weight will result in a higher interfacial velocity
4. Higher surface pressures will result in lower interfacial velocities

Acknowledgements

We wish to acknowledge the immense support given by both Dalhousie University and Cape Breton University Library Document Delivery sections, particularly for their timely delivery of literature resources without which the timely completion of this manuscript would not have been possible.

Nomenclature

C = constant of integration

constant of integration for no annular mud flow

D = depth of well at the time of the kick event

g = acceleration due to gravity

G = pit gain

d_h = hydraulic diameter of annular space

P = pressure in then flowing fluid

P_{surf} = surface pressure

Z_o = initial column thickness of kick fluid

P_{ib} = initial bottom hole pressure

P_{suf} = surface pressure

T_Z = temperature at annular position Z

j_m = mass flux

Z_o = initial column thickness of kick fluid

Z_Z = gas compressibility factor at annular position Z

T_{Z_0} = temperature at annular position Z_0

Z_{Z_0} = gas compressibility factor at annular position Z

V_D = interface velocity at well head

Greek Letters

β = parameter relating to temperature ratios

ρ_1 = density of component one

ρ_2 = density of component 2

φ = volume fraction of component 1

β = Temperature dependent parameter

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