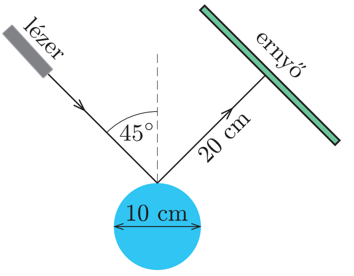
P. 5496. Az *ábrán* látható módon egy 10 cm átmérőjű, tükröződő felületű hengert egy 5 mm átmérőjű lézersugárral világítunk meg. A visszaverődő lézersugárra merőlegesen egy ernyőt helyezünk el úgy, hogy a visszaverődési pont és az ernyő között a távolság 20 cm. Milyen alakú és méretű fényfolt keletkezik az ernyőn?



(*5 pont*) Közli: *Széchenyi Gábor,* Budapest

P. 5497. Tekintsük a hidrogénatom Thomson-féle atommodelljét. A hidrogénatom sugara kb. 50 pm.

a) Hol lehet egyensúlyban az elektron?

b) Mekkora frekvenciával rezeg az elektron ezen egyensúlyi helyzet körül? A színkép milyen tartományába esik az ilyen frekvenciájú fény?

(*5 pont*) Közli: *Zsigri Ferenc,* Budapest

P. 5498. Egy α hajlásszögű, h magas éket könnyen gördülő, az ékkel együtt M tömegű kocsira rögzítettünk. Az ék alján egy $m \ll M$ tömegű test nyugszik. A kis testet úgy szeretnénk feljuttatni az ék tetejéig, hogy az éket állandó nagyságú, vízszintes erővel gyorsítjuk.

a) Legalább mekkora munkát kell végezzünk eközben, ha a súrlódás elhanyagolható?

b) A legkisebb munkavégzés esetén mekkora erővel hatunk az ékre és mennyi idő alatt emelkedik a kis test h magasságra?

Adatok: $h = 1$ m; $M = 1$ kg; $\alpha = 30°$.

(*6 pont*) Közli: *Holics László,* Budapest

Beküldési határidő: 2023. június 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS (Volume 73. No. 5. May 2023)

Problems in Mathematics

New exercises for practice – competition K (see page 286): **K. 769.** In Burger Burner Restaurant, it turned out that the soup is too salty. The chef decided to dilute it with the leftover soup from yesterday, which did not have enough salt in it. Given that 5 percent of the salty soup is salt while there is only 1.2 percent salt in the leftover soup, how much of each should the chef mix in order to obtain 72 decilitres of soup with a salt content of 3.48 percent? (Proposed by *K. A. Kozma,* Győr) **K. 770.** What is the ratio of the number of chessboard fields from which a knight may move to at least four fields to the number of fields from where the knight may move to eight fields? (Proposed by *B. Bíró,* Eger) **K. 771.** Frankie cut a rectangle into exactly nine squares. On inspection, he observed that the area of one square was 64 cm^2, the areas of two other squares were 16 cm^2, and the rest of them were 4 cm^2 each. What was the perimeter of the original rectangle? (Proposed by *K. A. Kozma,* Győr) **K/C. 772.** How many four-digit natural numbers are there in decimal notation in which the first three digits (from the left) are different, all four digits are prime numbers, but the four-digit number is not divisible by any of its digits? (Proposed by *B. Bíró,* Eger) **K/C. 773.** Is there a right-angled triangle in which the measures of the sides are integers, the lengths of exactly two sides are prime numbers and the area is also a prime number? (Proposed by *B. Bíró,* Eger)

New exercises for practice – competition C (see page 287): **Exercises up to grade 10: K/C. 772.** See the text at Exercises **K. K/C. 773.** See the text at Exercises **K. Exercises for everyone: C. 1768.** Show that the simultaneous equations $8x^3+27y^3=-6\cdot 5^3$, $\frac{3}{x}+\frac{2}{y}=\frac{xy}{5}$ have no solution if x, y are real numbers. (Proposed by *B. Bíró,* Eger) **C. 1769.** The orthocentre of an acute-angled triangle ABC is M, and $AB\geqslant BC\geqslant CA$ for the sides. The perpendicular bisector of line segment AM intersects side AC at D, and the perpendicular bisector of line segment BM intersects side BC at E. Find the angles of triangle ABC, given that points D, M, E are collinear. (Proposed by *B. Bíró,* Eger) **C. 1770.** Solve the equation $\sqrt{7+\frac{3}{\sqrt{x}}}=7-\frac{9}{x}$ over the set of real numbers. (Proposed by *B. Bíró,* Eger) **Exercises upwards of grade 11: C. 1771.** In isosceles right-angled triangle ABC, the midpoint of leg BC is D, and the point closer to vertex B that divides the hypotenuse in a one to two ratio is E. Prove that AD and CE are perpendicular. (Proposed by *B. Bíró,* Eger) **C. 1772.** How many at most three-digit numbers are there in decimal notation that become a palindrome number if converted to binary notation? (A number is called a palindrome if its digits read the same left to right and right to left.) (Proposed by *L. Koncz,* Budapest)

New exercises – competition B (see page 288): **B. 5318.** All positive divisors of a positive integer are written down on a sheet of paper. There are two numbers on the sheet that leave a remainder of 2 when divided by 8, and there are four numbers that leave a remainder of 4. How many numbers may there be on the sheet that leave a remainder of 6 when divided by 8? (*3 points*) (Proposed by *B. Hujter,* Budapest) **B. 5319.** Is it true that every acute-angled triangle has at least one altitude whose foot lies in the middle third of the side? (*3 points*) (Proposed by *B. Hujter,* Budapest) **B. 5320.** Given that $\frac{a_{n+3}}{a_{n+1}}+\frac{a_n}{a_{n+2}}=2$ for each term of a sequence a_n, and the first three terms are $a_1=1$, $a_2=4$ and $a_3=2$, prove that $\frac{2^{2021}}{a_{2023}}$ is an integer. (*5 points*) (Proposed by *A. Eckstein,* Temesvár) **B. 5321.** Show that the sum of the squares of the medians of a triangle is smaller than one and a half times the square of the semi-perimeter. (*4 points*) Proposed by *L. Németh,* Fonyód) **B. 5322.** Prove that if $\frac{\cos\alpha}{s-b}-\frac{\cos\beta}{s-a}=\frac{\cos\alpha-\cos\beta}{s-c}$ in a triangle, with conventional notations, then the triangle is right-angled or isosceles. (As usual, s denotes the semi-perimeter of the triangle.) (*5 points*) (Proposed by *G. Holló,* Budapest) **B. 5323.** We are playing the following game: arbitrary real numbers from the interval $[0,100]$ are written on 2023 cards. Then the cards are dropped in an urn, and one card is pulled out at random. If the number on the card equals 2/3 of the mean of the numbers on all the cards then we will win as much money as the number on the card pulled out. Otherwise we do not win anything. What numbers should be written on the cards so that the expected value of the money gained is a maximum? (*5 points*) (Proposed by *B. Dura-Kovács,* Garching) **B. 5324.** Arthur and Barbara are playing the following game: they take turns in writing down a digit, proceeding left to right, until they obtain a 2023-digit number. Arthur starts the game, with a nonzero digit. Arthur will win if the resulting number has a divisor of the form $1\overbrace{7\dots7}^{n\text{ pcs }7}$ $(n\geqslant 1)$. Otherwise Barbara wins. Which of them has a winning strategy? (*6 points*) (Based on the idea of *G. Kós,* Budapest) **B. 5325.** Determine all bounded convex polyhedra such that the planes of the faces divide the space into $c+e+\ell+1$ regions, where c, e and ℓ, respectively, stand for the number of vertices, edges and faces. (*6 points*) (Proposed by *V. Vígh,* Sándorfalva)

New problems – competition A (see page 290): **A. 854.** Prove that $\sum\limits_{k=0}^{n}\frac{2^{2^k}\cdot 2^{k+1}}{2^{2^k}+3^{2^k}}<4$ holds for all positive integers n. (Submitted by *Béla Kovács,* Szatmárnémeti) **A. 855.** In scalene triangle ABC the shortest side is BC. Let points M and N be chosen on sides AB and AC, respectively, such that $BM=CN=BC$. Let I and O denote the incenter and circumcentre of triangle ABC, and let D and E denote the incenter and circumcenter of triangle AMN. Prove that lines IO and DE intersect each other on the circumcircle of triangle ABC. (Submitted by *Luu Dong,* Vietnam) **A. 856.** In a rock-paper-scissors round robin tournament any two contestants play against each other ten times in a row. Each contestant has a favourite strategy, which is a fixed sequence of ten hands (for example, RRSPPRSPPS), which they play against all other contestants. At the end of the tournament it turned out that every player won at least one hand (out of the ten) against any other player. Prove that at most 1024 contestants participated in the tournament. (Submitted by *Dávid Matolcsi,* Budapest)

Problems in Physics

(see page 314)

M. 423. There is a 100-forint coin, initially at rest, on a horizontal tabletop. Send another 100-forint coin to slide along the tabletop such that it undergoes head on collision with the stationary one. Measure the distances covered by the two coins after the collision until they stop. From the measured data determine the *coefficient of restitution* which is a number to characterise the inelasticity of the collision. Does the coefficient of restitution depend on the relative velocity of the colliding objects?

G. 817. The James Webb space telescope orbits the so-called L_2 Lagrange point of the Sun-Earth system. This point is located 1.5 million km from Earth along the line connecting the centres of the Sun and the Earth, beyond the Earth. Imagine that you are exactly at the L_2 Lagrange point, looking towards the Sun. Do we need goggles? What do we see? **G. 818.** An open railway wagon is travelling on a horizontal straight track at a speed of v. With a cannon in the immediate vicinity of the railroad track, a projectile is fired at a speed of $2v$ at the moment when the end of the wagon passes the cannon. At what angle to the horizontal should the cannon fire its projectile so that it strikes the end of the wagon? How long after firing does the projectile fall back? (Neglect air resistance.) **G. 819.** The *figure* shows three tanks, to which drain tubes are attached, and they all contain the same liquid. Which tank will drain the fastest if the internal friction