(Proposed by *G. Kós,* Budapest) **B. 5314.** Let $S$ be an $n$-element set, and let $1 \leqslant k \leqslant n$ be an odd integer. What is the largest number of subsets of $S$ that can be selected so that the symmetric difference of no pair of subsets should have exactly $k$ elements? (*5 points*) (Proposed by *P. P. Pach,* Budapest) **B. 5315.** Consider a triangle $ABC$. Let $B'$ be a point on the extension of side $AB$ beyond $B$, and let $C'$ be the point on the extension of side $AC$ beyond $C$ such that $BB' = CC'$. Let $k$ and $k'$ denote the circumscribed circles of triangles $ABC'$ and $AB'C$, respectively. Prove that the common chord of $k$ and $k'$ lies on the angle bisector drawn from $A$. (*5 points*) (Proposed by *M. Hujter,* Budapest) **B. 5316.** Prove that if $0 < a, b < 1$ then $(a+b-ab)\left(a^b + b^a\right) > a+b$. (*6 points*) (Proposed by *M. Bencze,* Brassó) **B. 5317.** An ellipse lies in the closed positive orthant, its foci are $(x_1; y_1)$ and $(x_2; y_2)$, and it touches the coordinate axes at the points of abscissa $p$, and ordinate $q$, respectively. Show that the point $(p; q)$ is collinear with the origin and the centre of the ellipse, and calculate the numerical eccentricity of the ellipse. (*6 points*) (Proposed by *L. László,* Budapest)

**New problems – competition A** (see page 227): **A. 851.** Let $k$, $l$ and $m$ be positive integers. Let $ABCDEF$ be a hexagon that has a center of symmetry, and let its sidelengths be $AB = k$, $BC = l$ and $CD = m$. Let $f(k, l, m)$ denote the number of ways we can partition hexagon $ABCDEF$ into rhombi with unit sides and an angle of 120°. Prove that by fixing $l$ and $m$, there exists polynomial $g_{l,m}$ such that $f(k, l, m) = g_{l,m}(k)$ for every positive integer $k$, and find the degree of $g_{l,m}$ in terms of $l$ and $m$. (Submitted by *Zoltán Gyenes,* Budapest) **A. 852.** Let $(a_i, b_i)$ be pairwise distinct pairs of positive integers for $1 \leqslant i \leqslant n$. Prove that $(a_1 + a_2 + \cdots + a_n)(b_1 + b_2 + \cdots + b_n) > \frac{2}{9}n^3$, and show that the statement is sharp, i.e. for an arbitrary $c > \frac{2}{9}$ it is possible that $(a_1 + a_2 + \cdots + a_n)(b_1 + b_2 + \cdots + b_n) < cn^3$. (Submitted by *Péter Pál Pach,* Budapest, based on an OKTV problem) **A. 853.** Let points $A$, $B$, $C$, $A'$, $B'$, $C'$ be chosen in the plane such that no three of them are collinear, and let lines $AA'$, $BB'$, $CC'$ be tangent to a given equilateral hyperbole at points $A$, $B$ and $C$, respectively. Assume that the circumcircle of $A'B'C'$ is the same as the nine-point circle of triangle $ABC$. Let $s(A')$ be the Simson line of point $A'$ with respect to the pedal triangle of $ABC$. Let $A^*$ be the intersection of line $B'C'$ and the perpendicular of $s(A')$ through point $A$. Points $B^*$ and $C^*$ are defined in a similar manner. Prove that points $A^*$, $B^*$ and $C^*$ are collinear. (Submitted by *Áron Bán-Szabó,* Budapest)

## Problems in Physics

(see page 250)

**M. 422.** Hold a bar magnet close to a relatively large sheet of iron perpendicular to the sheet. Measure the magnetic force exerted on the bar magnet as a function of the distance from the metal plate.

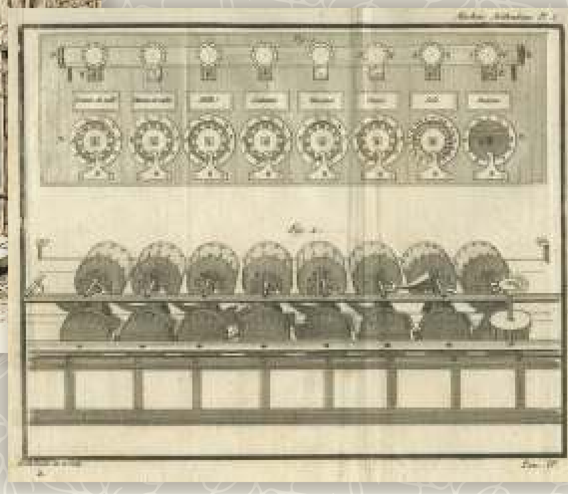
**G. 813.** From the top of a tower block, we took a series of photos of the traffic on the street next to the house. Two selected shots were taken with a time difference of 4/15 seconds, and show cars travelling at a constant speed. Estimate the speed of the cars relative to the roadway if the length of a white line segment of the white, dashed road marking, which divides the lanes of the street, is about 2 metres. **G. 814.** A heavy, open (railway) wagon is travelling on a horizontal, straight track at a speed of $v$. A light toy cannon on the wagon can fire projectiles at a speed of $2v$ with respect to the to the cannon. At what angle to the horizontal should the projectile be fired so that it falls back onto the wagon? How long after firing does the projectile fall back onto the wagon? (Neglect air drag.) **G. 815.** A lead ball of mass 2 g, and of density 11 300 $\frac{\mathrm{kg}}{\mathrm{m}^3}$ was frozen into a 100 g, 0 °C piece of ice of density 920 $\frac{\mathrm{kg}}{\mathrm{m}^3}$. The ice was put into water of temperature 0 °C. Due to the room temperature air around the ice-water system, 5 g of ice to melts in every minute. How long will it take for the ice to start sinking? **G. 816.** We have three resistors with resistance values of 1 kΩ, 2 kΩ and 4 kΩ. Two or three of these are connected in series and connected to 230 V voltage supply. In this way how many different values of voltage can we obtain across them, and what are they?

**P. 5481.** A vehicle starts from rest and accelerates uniformly. Point $P$, one of the outermost point on the rim of the vehicle wheel, is initially at its furthest position from the ground. By what factor does the acceleration of point $P$ increase after $n$ turns of the wheel? **P. 5482.** The thread of a simple pendulum of length $L$ is stretched horizontally and then released. When the thread of the pendulum becomes vertical, the pendulum bob collides perfectly elastically with another small body, which has the same mass as the pendulum bob, and which is initially on the edge of a table. After the collision, the body on the edge of the table is projected horizontally, i.e. it will move in a parabolic path. Where is the focus and the directrix of this parabola? **P. 5483.** Two point-like weights of mass $m$ and $M=2m$ are attached to the two endpoints of a diameter of a tire of radius $R$ and of negligible mass. The tire is placed on a frictionless table such that the plane of the tire is vertical, and initially the two weights are along the same vertical line (the heavier one is on the top). The tire is released from this unstable equilibrium state. *a*) What is the velocity of the centre of the tire when the weight of mass $M$ reaches the lowest point of its trajectory? *b*) In the case *a*), what is the force exerted on the table? **P. 5484.** A sample of diatomic gas is taken through the cyclic process which is a circle in the $p-V$ diagram, when appropriate units are used, and is shown in the *figure.* Using numerical methods, determine the efficiency of the heat engine which executes the above cyclic process. **P. 5485.** The vertices of a regular pentagon are connected along the sides by wires which have the same resistance, as shown in the *figure.* In another regular pentagon, we place wires along the diagonals to form a five-pointed star. (The wires are insulated and there are electrical joints only at the vertices of the pentagon.) The equivalent resistances measured between adjacent vertices of the two pentagons ($R_{AB}$ and $R_{PQ}$) are the same in the two connections. In which circuit will the equivalent resistance between the vertices of a diagonal ($R_{AC}$ or $R_{PR}$) be greater, and by what factor? **P. 5486.** The components of the circuit shown in the *figure* are ideal. Initially, one of the capacitors is charged to $q_0$, and the other capacitor is uncharged. *a*) What is the maximum current after closing switch K? *b*) How long after closing the switch does the current first reach its maximum value? **P. 5487.** The flat surface of a half-cylinder glass of refractive index $n$ is tin coated. The half-cylinder is illuminated horizontally with a laser beam as shown in the *figure.* At what value of $\alpha$ will the emerging light beam be exactly vertical? What should the minimum value of $n$ be for such a beam path to be possible? **P. 5488.** $\alpha$-particles are accelerated through a potential difference of $10^6$ V and then the particle beam enters perpendicularly into a region of uniform magnetic field of induction $B=1.5$ T and of width $d=7$ cm, as shown in the *figure.* *a*) At what angle are the particles deflected? *b*) How much time are the particles in the magnetic field? **P. 5489.** A rectangular frame was constructed with horizontal sides of length $a$ made of rigid, straight pieces of wires each of mass $m$, and vertical sides of length $b$ made of thin, negligible-mass threads. The frame was immersed into some dishwashing liquid, holding by one of the wires, and then it was taken out of it. The width of the resulting soap film was reduced to $d$ at the centre. What is the surface tension of the liquid? *Data:* $a=5$ cm, $b=8$ cm, $d=3.6$ cm, $m=2.6$ g.

# 400 éve született Blaise Pascal

Pascal hidrosztatikai kísérlete Rouenben

A mechanikus számológépének rajza




Pascalt ábrázoló 0 eurós emlékbankjegy

# **Matek az Utcán** – szerte az országban
2023.0**3.14**.

A ceglédi *Matek az Utcán* drónról fényképezve

Sapkás logikai feladat
Debrecenben

Matek Flashmob a Blaha Lujza téren
Budapesten – A π megmérése

Polidron labda építése

Logikai játék az aszfalton

Számolós kicsiknek Cegléden