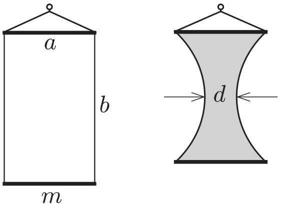
P. 5489. Egy olyan téglalap alakú keretet készítettünk, amelynek a hosszúságú vízszintes oldalai merev, egyenes, m tömegű drótszálak, b hosszúságú függőleges oldalai pedig vékony, elhanyagolható tömegű cérnaszálak.



A keretet az egyik drótszálnál fogva mosogatószeres oldatba mártottuk, majd kiemeltük. A kialakuló hártya mérete a közepénél d értékre csökkent. Mekkora a folyadék felületi feszültsége?

Adatok: $a = 5$ cm, $b = 8$ cm, $d = 3{,}6$ cm, $m = 2{,}6$ g.

(*6 pont*) *Varga István* (1952–2007) feladata

Áprilisi pótfeladat.* Ha szeretnénk kipróbálni, hogy milyen az ejtőernyős ugrás, akkor ezt az úgynevezett tandemugrással tehetjük meg. A tandemugrást bemutató képen az oktató felül helyezkedik el, alatta pedig az ejtőernyőzést kipróbáló érdeklődő látható.

Az érdeklődő és az oktató teste egymáshoz szorul, vagy éppen ellenkezőleg, távolodni akar egymástól, ha

a) az ejtőernyő még nincs nyitva, az ugrók „szabadon” zuhannak;

b) az ejtőernyő már huzamosabb ideig nyitva van?

❄

Beküldési határidő: 2023. május 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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Problems in Mathematics

New exercises for practice – competition K (see page 223): **K. 764.** In the Seventh Kingdom in the back of beyond, a week lasts one seventh as many days as on the Earth, a day consists of 42 hours, and there are 77 minutes in an hour and 33 seconds in a minute. How many seconds elapse during the course of two weeks there? (Proposed

* A feladat megoldása beküldhető, de nem számít bele a pontversenybe.

by *K. A. Kozma,* Győr) **K. 765.** The midpoint of side AB of a triangle ABC is D, and the midpoint of CD is E. Which point of the line segment CD should be marked F so that the sum of the areas of triangles AEC and BFC is exactly 40% of the area of triangle ABC? (Proposed by *B. Bálint,* Eger) **K. 766.** Alpha, Lambda and Zeta each have more than 1000 forints (HUF, Hungarian currency) on their bank accounts. Lambda's money equals 35 percent of Alpha's money, and Zeta's money equals $\frac{12}{7}$ of Lambda's money. How much money do Alpha, Lambda and Zeta have altogether if Zeta has 10 110 forints more than Lambda? (Proposed by *K. A. Kozma,* Győr) **K/C. 767.** The circle k passes through vertices A, B of a given square $ABCD$ in the plane, and touches the side CD. Let M denote the intersection of circle k and side BC which is different from B. Find the exact value of the ratio $\frac{CM}{BM}$. (Proposed by *I. Keszegh,* Révkomárom) **K/C. 768.** The number 2023 has exactly one digit of 0. How many four-digit positive odd numbers are there for which this property does not hold? (Proposed by *K. A. Kozma,* Győr)

New exercises for practice – competition C (see page 224): **Exercises up to grade 10: K/C. 767.** See the text at Exercises **K. K/C. 768.** See the text at Exercises **K. Exercises for everyone: C. 1763.** Prove that the number $4^{52}+52^{2023}+2023^{52}$ is divisible by 15. **C. 1764.** Solve the simultaneous equations $x(2x+6)(3x+5y)=64$; $2x^2+9x+5y=16$, where x, y are positive real numbers. (Proposed by *B. Bíró,* Eger) **C. 1765.** The base of a regular four-sided pyramid $ABCDE$ is the square $ABCD$, and each edge of the pyramid is 32 units long. A snail starts at vertex E and crawls to vertex A as follows: first moves along edge EA to the point P for which $EP=2$. Then continues to cross the face ABE and arrives at point Q of edge EB, where $EQ=4$. Then crosses the face BCE to reach that point R of edge EC for which $ER=8$, and continues along face CDE to point S on edge ED, with $ES=16$. Finally, it crawls on the surface of face DAE, from S to point A. What is the minimum distance covered by the snail altogether? (Proposed by *B. Bíró,* Eger) **Exercises upwards of grade 11: C. 1766.** Show that in every triangle (with conventional notations), $\sqrt{a\sin\alpha}+\sqrt{b\sin\beta}+\sqrt{c\sin\gamma}=\sqrt{(a+b+c)(\sin\alpha+\sin\beta+\sin\gamma)}$. (Proposed by *G. Holló,* Budapest) **C. 1767.** Given are 2 coins of 7, 3 coins of 17, 5 coins of 119, 7 coins of 289, 11 coins of 2023 and n coins of 1. Two coins are selected at random, and their values are multiplied together, and a result of 2023 is obtained. Find the value of n, given that the probability of obtaining that result is $\frac{12}{55}$. (Proposed by *O. Teleki,* Tököl)

New exercises – competition B (see page 225): **B. 5310.** A game of strategy is played by four teams on a map represented by an $n\times n$ square grid ($n\geqslant 3$). Every square field is either *sea* or *land.* The bases of the four teams are in the four corners of the map, on land. Given that there is one large connected region of sea on the map and no two bases are connected by a path on land, determine the minimum possible number of fields that represent the sea. (A path consists of adjacent fields. Two fields are adjacent if they have an edge in common. The region of sea is connected in this sense: any two sea squares can be connected by a path consisting of sea squares.) (*4 points*) (Proposed by *K. Williams,* Cambridge) **B. 5311.** Is it true that if the sine of each angle of a triangle is rational then the cosine of each angle is rational, too? (*3 points*) (Proposed by *M. Hujter,* Budapest) **B. 5312.** Let F_k denote the kth Fibonacci number ($F_1=F_2=1,\ F_{k+1}=F_k+F_{k-1}$). Prove that $2\sum_{k=1}^{n}F_k^2F_{k+1}=F_nF_{n+1}F_{n+2}$ for all positive integers n. (*3 points*) (Proposed by *M. Bencze,* Brassó) **B. 5313.** Triangle ABC is acute angled, and $AC<AB<BC$. The centre of the circumscribed circle is O, the orthocentre is M. The perpendicular bisector of side AB intersects line AM at point P, and the circle OMP intersects line BM again at a point Q, different from M. Prove that line BC is tangent to the circle ABQ. (*4 points*)

(Proposed by *G. Kós,* Budapest) **B. 5314.** Let S be an n-element set, and let $1 \leqslant k \leqslant n$ be an odd integer. What is the largest number of subsets of S that can be selected so that the symmetric difference of no pair of subsets should have exactly k elements? (*5 points*) (Proposed by *P. P. Pach,* Budapest) **B. 5315.** Consider a triangle ABC. Let B' be a point on the extension of side AB beyond B, and let C' be the point on the extension of side AC beyond C such that $BB' = CC'$. Let k and k' denote the circumscribed circles of triangles ABC' and $AB'C$, respectively. Prove that the common chord of k and k' lies on the angle bisector drawn from A. (*5 points*) (Proposed by *M. Hujter,* Budapest) **B. 5316.** Prove that if $0 < a, b < 1$ then $(a+b-ab)\left(a^b+b^a\right) > a+b$. (*6 points*) (Proposed by *M. Bencze,* Brassó) **B. 5317.** An ellipse lies in the closed positive orthant, its foci are $(x_1; y_1)$ and $(x_2; y_2)$, and it touches the coordinate axes at the points of abscissa p, and ordinate q, respectively. Show that the point $(p; q)$ is collinear with the origin and the centre of the ellipse, and calculate the numerical eccentricity of the ellipse. (*6 points*) (Proposed by *L. László,* Budapest)

New problems – competition A (see page 227): **A. 851.** Let k, l and m be positive integers. Let $ABCDEF$ be a hexagon that has a center of symmetry, and let its sidelengths be $AB = k$, $BC = l$ and $CD = m$. Let $f(k, l, m)$ denote the number of ways we can partition hexagon $ABCDEF$ into rhombi with unit sides and an angle of $120°$. Prove that by fixing l and m, there exists polynomial $g_{l,m}$ such that $f(k, l, m) = g_{l,m}(k)$ for every positive integer k, and find the degree of $g_{l,m}$ in terms of l and m. (Submitted by *Zoltán Gyenes,* Budapest) **A. 852.** Let (a_i, b_i) be pairwise distinct pairs of positive integers for $1 \leqslant i \leqslant n$. Prove that $(a_1+a_2+\dots+a_n)(b_1+b_2+\dots+b_n) > \frac{2}{9}n^3$, and show that the statement is sharp, i.e. for an arbitrary $c > \frac{2}{9}$ it is possible that $(a_1+a_2+\dots+a_n)(b_1+b_2+\dots+b_n) < cn^3$. (Submitted by *Péter Pál Pach,* Budapest, based on an OKTV problem) **A. 853.** Let points A, B, C, A', B', C' be chosen in the plane such that no three of them are collinear, and let lines AA', BB', CC' be tangent to a given equilateral hyperbole at points A, B and C, respectively. Assume that the circumcircle of $A'B'C'$ is the same as the nine-point circle of triangle ABC. Let $s(A')$ be the Simson line of point A' with respect to the pedal triangle of ABC. Let A^* be the intersection of line $B'C'$ and the perpendicular of $s(A')$ through point A. Points B^* and C^* are defined in a similar manner. Prove that points A^*, B^* and C^* are collinear. (Submitted by *Áron Bán-Szabó,* Budapest)

Problems in Physics

(see page 250)

M. 422. Hold a bar magnet close to a relatively large sheet of iron perpendicular to the sheet. Measure the magnetic force exerted on the bar magnet as a function of the distance from the metal plate.

G. 813. From the top of a tower block, we took a series of photos of the traffic on the street next to the house. Two selected shots were taken with a time difference of 4/15 seconds, and show cars travelling at a constant speed. Estimate the speed of the cars relative to the roadway if the length of a white line segment of the white, dashed road marking, which divides the lanes of the street, is about 2 metres. **G. 814.** A heavy, open (railway) wagon is travelling on a horizontal, straight track at a speed of v. A light toy cannon on the wagon can fire projectiles at a speed of $2v$ with respect to the to the cannon. At what angle to the horizontal should the projectile be fired so that it falls back onto the wagon? How long after firing does the projectile fall back onto the wagon? (Neglect