New problems – competition A (see page 98): **A. 845.** The incircle of triangle ABC is tangent to sides BC, AC, and AB at points D, E, and F, respectively. Let A' denote the point of the incircle for which circle $(A'BC)$ is tangent to the incircle. Define points B' and C' similarly. Prove that lines $A'D$, $B'E$ and $C'F$ are concurrent. (Proposed by *Áron Bán-Szabó,* Budapest) **A. 846.** Let n be a positive integer and let vectors $v_1, v_2, \ldots, v_n$ be given in the plain. A flea originally sitting in the origin moves according to the following rule: in the i^{th} minute (for $i = 1, 2, \ldots, n$) it will stay where it is with probability $1/2$, moves with vector v_i with probability $1/4$, and moves with vector $-v_i$ with probability $1/4$. Prove that after the n^{th} minute there exists no point which is occupied by the flea with greater probability than the origin. (Proposed by *Péter Pál Pach,* Budapest) **A. 847.** Let A be a given finite set with some of its subsets called *pretty.* Let a subset be called *small,* if it's a subset of a pretty set. Let a subset be called *big,* if it has a pretty subset. (A set can be small and big simultaneously, and a set can be neither small nor big.) Let a denote the number of elements of A, and let p, s and b denote the number of pretty, small and big sets, respectively. Prove that $2^a \cdot p \leqslant s \cdot b$. (Proposed by *András Imolay,* Budapest)

Problems in Physics

(see page 122)

M. 420. Fill several tubes having different diameter with rice. Measure the pressure as a function of height at the bottom of the rice column for each tube. Plot your results on a graph.

G. 805. Estimate the factor by which the pressure required to push garlic through a garlic press is greater than the atmospheric pressure. **G. 806.** In the circuit shown in the *figure,* the resistors R_1, R_2 and R_3 are known, as well as the current I_3 which flows through resistor R_3. Determine the *a*) values of the current, I_1 and I_2, through the other two resistors, *b*) the electromotive force of the battery. *c*) How much heat is dissipated in the whole system in a time of t? (*Data:* $R_1 = 20\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 40\ \Omega$, $I_3 = 2$ A, $t = 30$ s.) **G. 807.** From a height of 20 metres, three steel balls are projected one after the other in every second. The angle between the horizontal and the initial velocity of the first ball is 30° upwards, that of the third ball is 30° downwards, and the second ball is dropped without initial velocity. All three balls hit the ground at the same time. What were the initial velocities of the first and third steel balls? **G. 808.** A cylindrical container with a piston contains air of temperature 20 °C, and of relative humidity 30%. Keeping the temperature constant, to how many times of the original value must the volume of air in the container be changed to cause the water vapour in the container to begin to condense?

P. 5463. A rickety space probe "hovers" above the surface of an unknown planet, which has no atmosphere, at a height of $H = 225$ m. One after the other, two screws fall off. The second screw falls from the space probe just as the first has fallen 16 m. What is the distance between the two screws at the moment when the first one reaches the surface of the planet? **P. 5464.** Inclined planes of different angles of inclination are laid through the focus F of a parabola with vertical symmetry axis and opening downtranslwards. What is the angle of inclination of that inclined plane along which a point-like body, starting from the point F without initial velocity and sliding frictionlessly down, reaches the parabola in the shortest possible time? **P. 5465.** A heavy body of mass M is suspended on a light spring of spring constant D. The system is held at rest, and from a given moment the upper end of the spring is raised at a constant velocity v_0. Give the displacement of the body as

a function of time. **P. 5466.** On a damp spring morning, the temperature is 1 °C and the relative humidity is 80%. In a room the temperature is 20 °C, and the relative humidity is 40%. Does the humidity in the room increase or decrease when the room is ventilated? **P. 5467.** A heating filament is wound uniformly along a 20 cm long copper rod of cross section 3 cm^2. The rod has a suitable electrical insulation along its entire length. The rod is held vertically such that its lower end is just immersed into water in a glass containing melting ice as well, so it remains at a constant temperature of 0 °C. How many degrees Celsius will the other end of the rod heat over a sufficiently long period of time if the heating filament heats the copper rod with a power of 100 W? **P. 5468.** Travelling waves carry not only energy but also momentum. *a*) Analysing the units (using dimensional analysis), find the relationship between the energy and the momentum transferred in a wave. A 100 m^2 vertical wall surface receives a sound wave of 100 dB, which is reflected back perpendicularly as an echo of pressure level 60 dB. *b*) Estimate the force exerted on the wall by the reflected sound wave! *Hint:* the pressure level of sound of intensity I can be given in decibel units according to the following formula: $\beta = 10\ \mathrm{dB} \lg \frac{I}{I_0}$, where $I_0 = 10^{-12}\ \mathrm{W/m}^2$, which is called the threshold of hearing. Intensity is the amount of energy that passes through a unit surface, perpendicularly, in a second. **P. 5469.** A point-like body with a charge of $q = 4 \cdot 10^{-7}$ C and a mass of $m = 3$ g is at zero gravity, and is moving in the electric field of a fixed point charge of $Q = 6 \cdot 10^{-7}$ C. It starts from rest and its velocity increases to $v = 2$ m/s, while it covers a distance of $d = 0.8$ m. What was the initial distance between the two charges? **P. 5470.** Two identical converging lenses are placed opposite each other so that their focal points coincide. One lens is illuminated by a beam of monochromatic light of uniform energy flux density. The beam is parallel to the common principal axis of the lenses. The lenses are coated with an anti-reflection layer, so that the effects of light absorption and reflection inside the lenses are negligible. *a*) Determine the direction of the forces exerted on the lenses. *b*) Estimate the magnitude of the forces. *Data:* the focal length of each lens is 10 cm, their diameter is 5 cm, the wavelength of the light which is used for illuminating is 590 nm, the power received by the first lens is 1 W. **P. 5471.** Three identical cylinders of radius R and mass m are made from ice and they are all released without initial velocity from the position shown in the *figure.* The surface of the ice is smooth, so friction is negligible everywhere. *a*) Determine and sketch the kinetic energy of one of the lower ice cylinders as a function of the displacement of the upper cylinder. *b*) What is the speed at which the top ice cylinder hits the ground, and to what speed do the other two ice cylinders accelerate?

Problems of the 2022 Kürschák competition

1. A square is divided into 2022 rectangles. Consider the lines determined by the sides of all the rectangles. At most how many different lines can we get?

2. For the primes p, q having residue 3 modulo 4 the equation $x^2 - pqy^2 = 1$ is solvable with positive integers x, y. Prove that the equation $|px^2 - qy^2| = 1$ is also solvable with positive integers x, y.

3. Let n be a positive integer. Suppose that for the real numbers $a_{i,j}$ $(1 \leqslant i, j \leqslant n)$ we have $a_{i,j} + a_{j,i} = 0$ for all i, j (in particular, $a_{i,i} = 0$ for all i). Prove that

$$\frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{i,j} \right)^2 \leqslant \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j}^2.$$

When do we have equality?