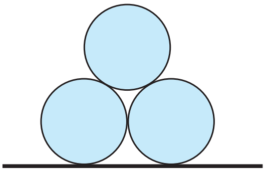

P. 5471. Három egyforma, R sugarú, m tömegű jéghengert készítünk, és azokat az *ábrán* látható helyzetből kezdősebesség nélkül elengedjük. A jég felülete nagyon síkos, emiatt a súrlódás mindenhol elhanyagolható.

a) Határozzuk meg és ábrázoljuk vázlatosan az egyik alsó jéghenger mozgási energiáját a felső henger elmozdulásának függvényében!

b) Mekkora sebességgel csapódik a felső jéghenger a talajhoz, és mekkora sebességre gyorsul fel a másik két jéghenger?

(*6 pont*) Közli: *Cserti József,* Budapest

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Beküldési határidő: 2023. március 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS (Volume 73. No. 2. February 2023)

Problems in Mathematics

New exercises for practice – competition K (see page 95): **K. 754.** Matt and Seb are playing tic-tac-toe. When Seb wins, he will get 3 candies from Matt. When Matt wins, he will get 2 candies from Seb. (There is no draw.) After 30 games, Seb had the same number of candies as initially. How many games did Seb win? **K. 755.** What is the maximum possible number of sides that a convex polygon may have if it has exactly 3 obtuse angles? Give an example of such a polygon. **K. 756.** A shop sells wooden cubes in two sizes: cubes of edge 1 cm and those with edges of 2 cm. The surfaces of the cubes are painted. In the case of the smaller cubes the paint represents 60% of the material costs, while the rest is the price of the wood. Both kinds of cube are made of the same type of wood and they are coated with a thin layer of paint with the same thickness. Labour costs are the same for each cube, independent of the size. Considering all costs, it costs the shop 830 forints (HUF, Hungarian currency) to manufacture 10 small cubes and 5 large ones. 5 small cubes and 15 large ones cost 1490 forints to manufacture. What is the labour cost of one cube? **K/C. 757.** Kate is planning to cut a 4×4 sheet of squared paper into pieces with scissors, cutting along grid lines only. Show that she can make exactly 11 different kinds of puzzle that is symmetrical in all four symmetry axes of the original square sheet. The *diagram* shows a possible puzzle. **K/C. 758** A sail is shaped like a right angled triangle and it has the red symbol of the boat class at a height such that $MA+AC=CB+BM$

(see the *figure*). If $BM = 7$ m and $CB = 5$ m, what is the height of the peak of the sail above the red mark?

New exercises for practice – competition C (see page 96): **Exercises up to grade 10: K/C. 757.** See the text at Exercises **K. K/C. 758.** See the text at Exercises **K. Exercises for everyone: C. 1753.** A long strip of paper is divided into squares. The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are written in the first ten squares, then the same numbers are written in the next ten squares, too, and so on. There are exactly 2030 squares numbered in this way. A token is placed in square 1. In each move, the token is moved as many squares ahead as the number written in the square it is standing on. What is the number in the square where the token is when its next move would get out off the paper strip of length 2030? (Proposed by *B. Bíró,* Eger) **C. 1754.** Squares $ABCD, BEFC$ and $EGHF$ are drawn next to each other in a plane. The foot of the perpendicular drawn from B to DE is K. Show that the points A, K, H are collinear. (Proposed by *B. Bíró,* Eger) **C. 1755.** What integers a, b and c satisfy the equality $a^2+b^2-8c=6$? (*Canadian problem*) **Exercises upwards of grade 11: C. 1756.** Solve the equation $4\cdot\cos\big(\pi\cdot\sin(\pi\cdot x)\big)=-5x^2+15x-\frac{61}{4}$ over the set of real numbers. (Proposed by *B. Bíró,* Eger) **C. 1757.** Johnny got a robot for Christmas, and he is trying it on a square carpet. The length of the sides of the square is 4 metres, and its vertices are A, B, C and D, in this order. Let P denote the interior point of square $ABCD$ which is exactly 1 metre away from both of sides AB and BC. The robot is initially standing at point P, and starts to move in a random direction. When it has covered 2 metres, it will stop. What is the probability that the robot will get off the carpet? (Proposed by *K. A. Kozma,* Győr)

New exercises – competition B (see page 97): **B. 5294.** Two altitudes of an acute-angled triangle ABC are AT_A and BT_B. The midpoint of AB is F, and the midpoint of T_AT_B is G. Prove that FG is perpendicular to T_AT_B. (*3 points*) (Proposed by *V. Vígh,* Sándorfalva) **B. 5295.** Find the largest integer k for which 1722 divided by k leaves a remainder of $2m$ and 2179 divided by k leaves a remainder of $3m$ (for some appropriate natural number $0\leqslant m<k/3$). (*3 points*) (Proposed by *K. A. Kozma,* Győr) **B. 5296.** A rook is moving from the bottom left corner of a 8×8 chessboard to the top right corner. It starts by moving to the right, then up, and so on, right and up alternatingly. How many different sequences of moves are there? (*4 points*) (Proposed by *B. Hujter,* Budapest) **B. 5297.** In a triangle ABC, $\angle BAC=2\angle CBA$. Let A', B', C' denote interior points of sides CA, AB and BC, respectively, such that triangle $A'B'C'$ is similar to triangle ABC. Show that the angle bisectors of angles BAC and $B'A'C'$ intersect each other on the line segment $B'C'$. (*4 points*) (Proposed by *G. Kós,* Budapest) **B. 5298.** Solve the following system of equations over the set of real numbers: $y+yx^2-2x=0$, $z+zy^2-2y=0$, $x+xz^2-2z=0$. (*5 points*) (*American problem*) **B. 5299.** There is a flea sitting at each of points $1, 2, 3$ of the number line. If one flea is at point a and another is at point b, then the flea at a may jump to the point $2b-a$. Is it possible for the fleas to end up at points $2^{100}, 3^{100}, 2^{100}+3^{100}$ with a finite sequence of such steps? (*6 points*) (Proposed by *P. P. Pach,* Budapest) **B. 5300.** Let T be a regular tetrahedron of unit edge. Inscribe a cube in T such that exactly two vertices of the cube lie on each face of T, as shown in the *figure* (the dashed lines are parallel to the appropriate edges of the tetrahedron). What is the volume of the cube? (*5 points*) (Proposed by *V. Vígh,* Sándorfalva) **B. 5301.** Given that the sum of the reciprocals of ten distinct positive integers is 1, prove that each of them is smaller than 10^{1000}. (*6 points*) (Proposed by *V. Vígh,* Sándorfalva)

New problems – competition A (see page 98): **A. 845.** The incircle of triangle ABC is tangent to sides BC, AC, and AB at points D, E, and F, respectively. Let A' denote the point of the incircle for which circle $(A'BC)$ is tangent to the incircle. Define points B' and C' similarly. Prove that lines $A'D$, $B'E$ and $C'F$ are concurrent. (Proposed by *Áron Bán-Szabó,* Budapest) **A. 846.** Let n be a positive integer and let vectors $v_1, v_2, \dots, v_n$ be given in the plain. A flea originally sitting in the origin moves according to the following rule: in the i^{th} minute (for $i = 1, 2, \dots, n$) it will stay where it is with probability 1/2, moves with vector v_i with probability 1/4, and moves with vector $-v_i$ with probability 1/4. Prove that after the n^{th} minute there exists no point which is occupied by the flea with greater probability than the origin. (Proposed by *Péter Pál Pach,* Budapest) **A. 847.** Let A be a given finite set with some of its subsets called *pretty.* Let a subset be called *small,* if it's a subset of a pretty set. Let a subset be called *big,* if it has a pretty subset. (A set can be small and big simultaneously, and a set can be neither small nor big.) Let a denote the number of elements of A, and let p, s and b denote the number of pretty, small and big sets, respectively. Prove that $2^a \cdot p \leqslant s \cdot b$. (Proposed by *András Imolay,* Budapest)

Problems in Physics

(see page 122)

M. 420. Fill several tubes having different diameter with rice. Measure the pressure as a function of height at the bottom of the rice column for each tube. Plot your results on a graph.

G. 805. Estimate the factor by which the pressure required to push garlic through a garlic press is greater than the atmospheric pressure. **G. 806.** In the circuit shown in the *figure,* the resistors R_1, R_2 and R_3 are known, as well as the current I_3 which flows through resistor R_3. Determine the *a*) values of the current, I_1 and I_2, through the other two resistors, *b*) the electromotive force of the battery. *c*) How much heat is dissipated in the whole system in a time of t? (*Data:* $R_1 = 20\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 40\ \Omega$, $I_3 = 2$ A, $t = 30$ s.) **G. 807.** From a height of 20 metres, three steel balls are projected one after the other in every second. The angle between the horizontal and the initial velocity of the first ball is 30° upwards, that of the third ball is 30° downwards, and the second ball is dropped without initial velocity. All three balls hit the ground at the same time. What were the initial velocities of the first and third steel balls? **G. 808.** A cylindrical container with a piston contains air of temperature 20 °C, and of relative humidity 30%. Keeping the temperature constant, to how many times of the original value must the volume of air in the container be changed to cause the water vapour in the container to begin to condense?

P. 5463. A rickety space probe "hovers" above the surface of an unknown planet, which has no atmosphere, at a height of $H = 225$ m. One after the other, two screws fall off. The second screw falls from the space probe just as the first has fallen 16 m. What is the distance between the two screws at the moment when the first one reaches the surface of the planet? **P. 5464.** Inclined planes of different angles of inclination are laid through the focus F of a parabola with vertical symmetry axis and opening downtranslwards. What is the angle of inclination of that inclined plane along which a point-like body, starting from the point F without initial velocity and sliding frictionlessly down, reaches the parabola in the shortest possible time? **P. 5465.** A heavy body of mass M is suspended on a light spring of spring constant D. The system is held at rest, and from a given moment the upper end of the spring is raised at a constant velocity v_0. Give the displacement of the body as