Ha a hasábot elengedjük, a rendszer mozgásba jön, és a kocka elcsúszik a hasábon. A két test közötti csúszás azt a pillanatot követően szűnik meg, amikor a hasáb gyorsulása nullává válik.

a) Mennyi ideig csúszott a kocka a hasábon?

b) Legalább mekkora az ℓ távolság, ha a leírt mozgás létrejöhetett?

c) Mekkora távolsággal csúszott el a kocka a hasábon?

(*6 pont*) Közli: *Wiedemann László,* Budapest

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Problems in Mathematics

New exercises for practice – competition K (see page 29): **K. 749.** Aladdin found five coins in a box. One of them is a counterfeit coin, and the monkey Abu is the only one who knows which. If Aladdin selects three coins and gives one of them to Abu, then Abu will tell him whether there is a counterfeit one among the other two. Whenever Abu gets a real coin, he will tell the truth, but he will lie if he gets a counterfeit coin. Is it possible to identify the counterfeit coin with at most three questions? (Based on the idea of *S. Róka,* Nyíregyháza) **K. 750.** When Pete walks to the school, he always has the same speed. Sometimes he is in a hurry and then he runs, with twice the walking speed. Yesterday he walked the first third of the distance to the school and then covered the rest of the distance running. Today, he walked 6 minutes longer than the time spent running. How many minutes longer did it take him today to get to the school than yesterday? **K. 751.** We have five chocolate trufles, all of them look alike. However, three of them weigh 20 g each, one weighs 19 g, and one weighs 21 g. We want to identify the 19-g trufle with the help of an equal-arm balance only. Prove that it is possible to do it by using the balance three times, but less than three is not enough. **K/C. 752.** There are k ways to select at least two of the numbers 10, 11, 12, 13, 14, 15, 16, 17 such that their sum is divisible by 3. In how many ways is it possible to select at least two of the numbers 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 such that their sum is divisible by 3? Express your answer in terms of k. (Two selections are considered different if they do not consist of the same set of numbers.) **K/C. 753.** Points B, C, D and E lie on one arm of an angle of vertex A, and points F, G, H and I lie on the other arm. Given that $AB=BG=GD=DI=IE=EH=HC=CF=FA$ (see the *figure*), show that the triangles CEH and IGD are equilateral.

New exercises for practice – competition C (see page 30): **Exercises up to grade 10: K/C. 752.** See the text at Exercises **K. K/C. 753.** See the text at Exercises **K. Exercises for everyone: C. 1748.** Show that the shortest side of a cyclic quadrilateral inscribed in a unit circle cannot be longer than $\sqrt{2}$. (*Canadian problem*) **C. 1749.** Calculate the exact value of $\sqrt[3]{K}$, where K denotes the product of all positive factors of 2025. (Proposed by *K. A. Kozma,* Győr) **C. 1750.** M and N are the common points of circles k_1, centred at O_1 and k_2, centred at O_2. A secant drawn through point M intersects circle k_1 at point A and circle k_2 at point B such that A lies outside circle k_2 and B lies outside circle k_1. Lines AO_1 and BO_2 intersect at point P. Points N and P lie in the same half plane determined by line O_1O_2. Show that P lies on the circumscribed circle of triangle O_1NO_2. (Proposed by *B. Bíró,* Eger) **Exercises upwards of grade 11: C. 1751.** Let a and b denote positive real numbers such that $a^2+b^2=\frac{2}{9}$. Prove that $\frac{1}{2-3a}+\frac{1}{2-3b}\geqslant 2$. (Proposed by *G. Szmerka,* Budapest) **C. 1752.** There are six people waiting in a line. It is taking very long, so they decide to play. They select a permutation of the six positions at random, and perform it three times successively. (A permutation is a rule of obtaining a new order by assigning a (possibly) new position to everyone. For example they may chose the following rule: the person at position 1 goes to position 3, the one at position 2 goes to position 1, the third goes to position 2, the fourth goes to position 6, the fifth stays in place, and the sixth goes to position 4.) What is the probability that at least one of them will be standing in their initial position at the end? (Proposed by *K. A. Kozma,* Győr)

New exercises – competition B (see page 31): **B. 5286.** What is the smallest positive integer n for which the number $\underbrace{11\ldots1}_{n}$ (in decimal notation) is divisible by the number $\underbrace{33\ldots3}_{100}$? (*3 points*) (*Brasilian problem*) **B. 5287.** Two circles touch each other externally. The line passing through the centres of the circles intersects the circles again at points A and B. The points of tangency on one of the common external tangents of the circles are P and Q, respectively. Prove that the lines AP and BQ intersect on the common internal tangent of the circles. (*3 points*) (Proposed by *I. Á. Molnár,* Miskolc) **B. 5288.** Two players are playing the following game on a 8×8 chessboard. They take turns in selecting one side of a field of the chessboard, separating two adjacent fields of the board, and colouring it yellow. The player who is first forced to create a closed yellow polygon will lose the game. Which player has a winning strategy? (*4 points*) (Based on an *American competition problem*) **B. 5289.** Let a, b, c and d denote non-negative real numbers such that $a+b+c+d=1$. Prove that $\frac{1}{a^2+1}+\frac{1}{b^2+1}+\frac{1}{c^2+1}+\frac{1}{d^2+1}\geqslant\frac{7}{2}$. (*5 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5290.** Solve the following equation over the set of positive integers: $3^n+4^n+\cdots+(n+2)^n=(n+3)^n$. (*6 points*) (Proposed by *T. Káspári,* Paks) **B. 5291.** The centre of the inscribed circle of a triangle ABC is I, and the centre of its circumscribed circle is O. Given that $\angle OIA=90^\circ$, $AI=89$ and $BC=160$, what is the area of the triangle? (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5292.** Given a circle k and points P and Q in its interior, construct* a circle through points P and Q that intersects the circle k at two diametrically opposite points. Depending on the position of points P and Q, how many solutions will the problem have? (*5 points*) (Proposed by *G. Kató,* Kápolnásnyék) **B. 5293.** Let p denote a prime number. What is the largest number

* Write down the steps of the construction (basic construction steps like bisecting an angle, reflecting over a line, etc. do not need to be described in detail) and explain why the method is correct. It is not required to perform the construction on paper.

of polynomials with integer coefficients such that there are no pairs of polynomials such that the value of their difference is divisible by p^2 at every integer? (*6 points*) (Proposed by *P. P. Pach,* Budapest)

New problems – competition A (see page 33): **A. 842.** n people live in a town, and they are members of some clubs (residents can be members of more than one club). No matter how we choose some (but at least one) clubs, there is a resident of the town who is the member of an odd number of the chosen clubs. Prove that the number of clubs is at most n. (Proposed by *Dömötör Pálvölgyi,* Budapest) **A. 843.** Let N be the set of those positive integers n for which $n \mid k^k - 1$ implies $n \mid k - 1$ for every positive integer k. Prove that if $n_1, n_2 \in N$, then their greatest common divisor is also in N. **A. 844.** The inscribed circle of triangle ABC is tangent to sides BC, AC and AB at points D, E and F, respectively. Let E' be the reflection of point E across line DF, and F' be the reflection of point F across line DE. Let line EF intersect the circumcircle of triangle $AE'F'$ at points X and Y. Prove that $DX = DY$. (Proposed by *Márton Lovas,* Budapest)

Problems in Physics

(see page 57)

M. 419. Cut out approximately 2 cm wide strips from a sheet of A4 copy paper, parallel to its longer and to its shorter side. Narrow the middle third of the paper strips along the curved lines as shown in the *figure.* Measure the tensile strength of the paper. (Express the tensile strength in MPa units). Is there a difference between the tensile strength of the longer and shorter strips?

G. 801. Two men, each having a mass of 72 kg, are standing on skis on the horizontal ground. The pressure exerted on the snow by the first man is 0.02 bar, and the pressure exerted by the second one is 0.05 bar. The second man is also carrying a 5 kg backpack on his back. *a*) What is the surface area of the skis, which is in contact with the snow? *b*) What will the pressure exerted by the second man be if he drops his backpack and stands on one leg only? **G. 802.** Approximately how many minutes later does the solar noon happen in Sopron than in Mátészalka? **G. 803.** The speed of an object travelling in a straight line with constant deceleration decreases to half of its initial value when it reaches the end of a straight path. What percentage of its initial speed is lost when it reaches the midpoint of the path? **G. 804.** A wooden block of density 0.6 kg/dm^3 is floating in a large pool. It has a height of 40 cm, the length of its base is 80 cm and the width of its base is 30 cm. The block is fixed to the bottom of the pool by a spring, having an unstretched length of 50 cm, as shown in the *figure.* *a*) What is the height of that part of the block, which is not under the water if the depth of the water is 90 cm and the spring constant of the spring is 1440 N/m? *b*) At least how much work is required to raise the block completely above the surface of the water if the spring does not break?

P. 5454. After repairing the tires of cars, the wheels have to be balanced. The wheels are placed to a balancing machine which spins them at very high speed, and measures the imbalance of the wheel. Then small wheel weights are mounted to the appropriate positions in order to balance the wheel. A wheel is rotated at a constant angular acceleration from rest. At a certain moment the speed of the valve cap, which is at a distance of $R = 20$ cm from the axle, is $v = 1$ m/s, and its acceleration is twice as big as it was when the wheel was started to move. *a*) How much time elapsed from the start? *b*) What was the acceleration of the valve cap, at the start of the rotation? *c*) How much distance did the valve cap cover during its rotation? **P. 5455.** A point-like object is given an initial horizontal velocity