P. 5441. Egy fémdrótból kört formáztunk, és ugyanabból a drótból az egyik húrt is szeretnénk elkészíteni a kör két pontja közé. Hol fusson a húr, hogy a lehető legnagyobb legyen az eredő ellenállás a húr két végpontja között, és mekkora lesz az eredő ellenállás ebben az esetben? Jelölje R a sugárhosszúságú drót ellenállását.

(*4 pont*) Közli: *Gáspár Merse Előd,* Budapest

P. 5442. Egy eredetileg nyugvó atommag 20 kV potenciálkülönbség befutása után a haladási irányára merőleges, 1,0 T indukciójú homogén mágneses mezőbe kerül. A mágneses mezőt egy, a részecske haladási irányára merőleges sík választja el az erőtérmentes tartománytól. A részecske $3{,}3\cdot 10^{-8}$ s múlva lép ki a mágneses mezőből. Melyik atommagról van szó?

(*4 pont*) Közli: *Tornyos Tivadar Eörs,* Budapest

P. 5443. A KCl lapcentrált kockarendszerben kristályosodik, és a rácsállandója 628 pm. Legfeljebb mekkora lehet a röntgenfény hullámhossza, hogy létrejöhessen Bragg-reflexió az elemi cella testátlóira merőleges rácssíkokon? (Lásd *A röntgenszórás, más néven Bragg-reflexió* c. cikket lapunk 489. oldalán.)

(*4 pont*) Közli: *Woynarovich Ferenc,* Budapest

P. 5444. Egy vékony, hosszú, függőleges, szigetelőrúdon súrlódásmentesen mozoghat egy kicsiny töltött golyócska. Ha egy ezzel azonos töltésű, ugyancsak kicsiny testet helyezünk a rúd tövébe, a mozgó golyó h_0 magasságban lesz egyensúlyban. Milyen messzire távolíthatjuk el a rúdtól vízszintes irányba az alsó testet úgy, hogy a rúdon lévő golyó még egyensúlyban lehessen valahol? Milyen magasan van ez a hely?

(*6 pont*) *Varga István* (1952–2007) feladata nyomán

Beküldési határidő: 2022. december 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS (Volume 72. No. 8. November 2022)

Problems in Mathematics

New exercises for practice – competition K (see page 480): **K. 739.** Phil made the following observations throughout a certain period in autumn: 1. During that period, there were 11 days when it rained. 2. A rainy morning was always followed by a sunny afternoon. 3. Altogether, there were 9 sunny mornings and 12 sunny afternoons. How many days were there when it did not rain at all? **K. 740.** In how many different ways is it possible to tile a 3×12 rectangle with twelve 1×3 rectangles? **K. 741.** Starting with the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$, in each step two numbers are chosen and increased by 1. Is it possible to achieve that each number is 10? **K/C. 742.** Danny is learning the alphabet. He has successfully named the first eight letters (A, B, C, D, E, F, G, H), but the order of the letters was not entirely correct. Only five of the eight letters were listed in the right position (i.e. in the position where it occurs in the alphabet). How many such orders of the eight letters are there? **K/C. 743.** The midpoint of side BC of a rectangle $ABCD$ is E, and F is the point lying closer to D which divides side CD in a $2:1$ ratio. The midpoint of line segment AE is G, and H is the point lying closer to E which divides line segment EF in a $2:1$ ratio. What fraction is the area of triangle FGH of the area of rectangle $ABCD$?

New exercises for practice – competition C (see page 480): **Exercises up to grade 10: K/C. 742.** See the text at Exercises **K. K/C. 743.** See the text at Exercises **K. Exercises for everyone: C. 1738.** A natural number is called *balanced,* if the number of digits in its representation in decimal notation equals the number of different prime factors it has. For example, 21 is balanced, but 42 is not. Is it true that there are infinitely many *balanced* numbers? (Proposed by *K. A. Kozma,* Győr) **C. 1739.** Define the following functions on the largest possible subset of the set of real numbers: $f(x)=\sqrt{x+5}$, $g(x)=\frac{-2x+8}{5}$ and $h(x)=[x+3]$ (here $[a]$ denotes the integer part of the real number a, that is, the greatest integer which is not greater than a). Find the common points of the graphs of the three functions. (Proposed by *B. Bíró,* Eger) **C. 1740.** P is an interior point of side CD of a parallelogram $ABCD$, and Q is an interior point of side AB parallel to CD. The intersection of line segments PA and QD is M, and the intersection of line segments PB and QC is N. Assume that $MN \nparallel AB$, and MN intersects the line of CD at point X, and the line of AB at point Y. Prove that $DX=BY$. (*American competition problem*) **Exercises upwards of grade 11: C. 1741.** The diagonals AC and BD of a convex quadrilateral $ABCD$ intersect at M. Is it possible that the areas of triangles ABM, BCM, CDM and DAM, in this order, are four consecutive terms of *a*) an arithmetic sequence; *b*) a geometric sequence? (Proposed by *B. Bíró,* Eger) **C. 1742.** Consider the following functions (defined on the largest possible subset of the set of real numbers): $f_0(x)=\frac{1}{1-x}$, and $f_n(x)=f_0\big(f_{n-1}(x)\big)$, for all positive integers n. Calculate the value of $f_{2022}(2022)$. (*Canadian problem*)

New exercises – competition B (see page 482): **B. 5270.** n^2 regular triangles of unit side are used to make a large regular triangle of side n units. The small triangles are coloured alternately dark and light. The numbers $1, 2, 3, \ldots, n^2$ are written in the triangles, as shown in the *figure.* What is the sum of the numbers in the dark triangles? (*3 points*) (Proposed by *L. Németh,* Fonyód) **B. 5271.** ABC is an isosceles right angled triangle with the right angle lying at vertex C. A', B' and C' are interior points of sides AB, BC and CA, respectively, such that triangle $A'B'C'$ is similar to ABC. Show that the midpoint of the side AB, the midpoint of line segment $A'B'$, and point C are collinear. (*3 points*) (Proposed by *E. Hajdu,* Sopron and *M. Hujter,* Budapest) **B. 5272.** A flea starts out from point (a, b) of the coordinate plane, where a, b are positive integers. With each jump, the flea will move one unit to the left or downwards. It keeps jumping until it reaches either the x axis or the y axis. What fraction of the possible sequences of jumps terminate on the x axis? (*4 points*) (Based on the idea of *D. Melján,* Kecskemét) **B. 5273.** D is a point on side AB of an equilateral triangle ABC, and E is a point on side BC such that $\angle BCD=45^\circ$ and $\angle CDE=30^\circ$. Show that $BE=2AD$. (*4 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5274.** The product of the positive integers $a<b$ is a perfect square. Show that there is a positive integer x such that $a \leqslant x^2 \leqslant b$. (*5 points*) (Proposed by *S. Róka,*

Nyíregyháza) **B. 5275.** Is there an irrational number a such that $a^{\sqrt{3}}$ is rational? (*5 points*) (Proposed by *B. Hujter,* Budapest) **B. 5276.** Prove that there exist infinitely many positive integers k for which the sum of the digits of 2^k is *a*) smaller; *b*) greater than the sum of the digits of 2^{k+1}. (*6 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5277.** The centre of the inscribed circle of triangle ABC is I. The midpoint of the circular arc BCA is F, and line FI intersects the circumscribed circle again at point M. Show that line CM passes through the external centre of similitude of the inscribed circle and the circumscribed circle. (*6 points*) (Proposed by *G. Kós,* Budapest)

New problems – competition A (see page 483): **A. 836.** For every $i \in \mathbb{N}$ let A_i, B_i and C_i be three finite and pairwise disjoint subsets of $\mathbb{N}$. Suppose that for every partition of $\mathbb{N}$ consisting of sets A, B and C there exists $i \in \mathbb{N}$ such that $A_i \subset A$, $B_i \subset B$ and $C_i \subset C$. Prove that there also exists a finite $S \subset \mathbb{N}$ such that for every partition of $\mathbb{N}$ consisting of sets A, B and C there exists $i \in S$ such that $A_i \subset A$, $B_i \subset B$ and $C_i \subset C$. (Submitted by *András Imolay,* Budapest) **A. 837.** Let all the edges of tetrahedron $A_1A_2A_3A_4$ be tangent to sphere S. Let a_i denote the length of the tangent from A_i to S. Prove that $\left(\sum_{i=1}^{4} \frac{1}{a_i}\right)^2 > 2\left(\sum_{i=1}^{4} \frac{1}{a_i^2}\right)$. (Submitted by *Viktor Vígh,* Szeged) **A. 838.** Sets $X \subset \mathbb{Z}^+$ and $Y \subset \mathbb{Z}^+$ are called comradely, if every positive integer n can be written as $n = xy$ for some $x \in X$ and $y \in Y$. Let $X(n)$ and $Y(n)$ denote the number of elements of X and Y, respectively, among the first n positive integers. Let $f : \mathbb{Z}^+ \to \mathbb{R}^+$ be an arbitrary function that goes to infinity. Prove that one can find comradely sets X and Y such that $\frac{X(n)}{n}$ and $\frac{Y(n)}{n}$ goes to 0, and for all $\varepsilon > 0$ exists $n \in \mathbb{Z}^+$ such that $\frac{\min\{X(n),Y(n)\}}{f(n)} < \varepsilon$.

Problems in Physics

(see page 506)

M. 417. Make a chain from 50 paper clips. Hold the chain vertically at one of its ends such that the other end just touches the table. Drop the chain several times in succession, and measure each time the maximum size of the tangled chain and the distance between the two ends of the chain. Calculate the mean and standard deviation of the measured values. Compare these with the total length of the chain when stretched.

G. 793. At a pressure of 1000 hPa a force equivalent to the weight of a 15 ton object is distributed over the body of a human being. *a*) what is the surface area of the body of the person? *b*) what is this weight at the highest point of the High Tatra Mountains? **G. 794.** The cross-section of a U-shaped tube is 1.5 cm^2. The tube is filled with mercury such that there is high enough mercury in both arms of the tube. 0.1 dl of water is poured on the mercury in one arm of the tube. In which arm and by what distance will the surface of the liquid be higher? **G. 795.** The angle between two plane mirrors is 60°. At a distance of 30 cm from the intersection of the two mirrors, a ray of light is incident on one of the mirrors, its angle of incidence is 30°. What is the minimum time that it takes for the reflected light to travel from one mirror to the other? **G. 796.** An ozone generator produces 5 g of ozone per hour by corona discharge and delivers it by a fan to the surface, which is to be disinfected. *a*) How many ozone molecules are produced in one hour? *b*) The manual recommends 30 minutes to disinfect a surface of area 28 m^2. The air is clean and dust-free, so the generated ozone will only decompose on the surface. Estimate the number of ozone molecules per bacterium on a surface area of 10 square microns.

P. 5436. Two cars, which can be considered to be point-like, are travelling at constant speed, each on a straight motorway towards the intersection of the motorways. (The angle