lines a, b and c (where the usual convention is used: points B and C lie on line a, and so on). Construct the point P on the circumcircle of the triangle for which the point P_a divides the line segment P_bP_c in a ratio $p:q$, where P_x is the orthogonal projection of the point P onto the line x. (*5 points*) **B. 5268.** Let I denote the incenter of the triangle ABC. Let P denote an arbitrary interior point of the triangle on the circle ABI. The reflection of the line AP about the line AI intersects the circle ABI at a point Q different from the point A. Prove that $CP=CQ$. (*6 points*) (Proposed by *Szilveszter Kocsis,* Budapest) **B. 5269.** Let $p\geqslant 19$ be an odd integer, and color the numbers $0,1,\ldots,p-1$ with two colors. For $1\leqslant i\leqslant p$, let x_i denote a random element of the set $\{0,1,\ldots,p-1\}$ (the choices are independent, and have uniform distribution). Prove that the probability of the event that $x_1,\ldots,x_p$ have the same color and p divides $x_1+\cdots+x_p$ is at least $3/(2^p p)$. (*6 points*) (Proposed by *Péter Pál Pach,* Budapest)

New problems – competition A (see page 419): **A. 833.** Some lattice points in the Cartesian coordinate system are colored red, the rest of the lattice points are colored blue. Such a coloring is called *finitely universal,* if for any finite, non-empty $A\subset\mathbb{Z}$ there exists $k\in\mathbb{Z}$ such that the point (x,k) is colored red if and only if $x\in A$. *a*) Does there exist a finitely universal coloring such that each row has finitely many lattice points colored red, each row is colored differently, and the set of lattice points colored red is connected? *b*) Does there exist a finitely universal coloring such that each row has a finite number of lattice points colored red, and both the set of lattice points colored red and the set of lattice points colored blue are connected? A set H of lattice points is called connected if, for any $x,y\in H$, there exists a path along the grid lines that passes only through lattice points in H and connects x to y. (Submitted by *Anett Kocsis,* Budapest) **A. 834.** Let $A_1A_2\ldots A_8$ be a convex cyclic octagon, and for $i=1,2\ldots,8$ let $B_i=A_iA_{i+3}\cap A_{i+1}A_{i+4}$ (indices are meant modulo 8). Prove that points $B_1,\ldots,B_8$ lie on the same conic section. **A. 835.** Let $f^{(n)}(x)$ denote the n^{th} iterate of function f, i.e $f^{(1)}(x)=f(x)$, $f^{(n+1)}(x)=f\Big(f^{(n)}(x)\Big)$. Let $p(n)$ be a given polynomial with integer coefficients, which maps the positive integers into the positive integers. Is it possible that the functional equation $f^{(n)}(n)=p(n)$ has exactly one solution f that maps the positive integers into the positive integers? (Submitted by *Dávid Matolcsi* and*Kristóf Szabó,* Budapest)

Problems in Physics

(see page 442)

M. 416. Hang a small object on the end of a door handle and gradually increase the load, with placing additional bodies on the load. Measure the angular position of the handle in equilibrium as a function of the mass hung on it. After reaching the maximum possible angle, gradually reduce the load and measure the angular position of the handle as a function of mass. Plot your data on the same graph.

G. 789. Estimate how much your body mass decreases while you sleep for eight hours peacefully! Use the following data: • During one breath 0.5 l of air is exchanged. • We breathe 15 times in a minute. • The carbon dioxide content of the exhaled air is 5 V/V%. • The exhaled air contains 6 V/V% water vapour. **G. 790.** The average consumption of a car is 6 litres/100 km. On a completely empty, winding two-lane road the car can travel 300 km. The total length of the curves on this road segment is 50 km, the radius of the curves is on average 1 km and the width of the lanes is 4 m. Estimate the reduction in fuel consumption if a careless driver makes every turn on the inside curve. **G. 791.** A juggling theoretical physicist invents the following stunt. He puts n perfectly elastic balls on top of each other with very small gaps between them. He drops the ball tower onto a hard surface, where the balls arrive at a speed of v. After a series of momentary collisions, all the balls except the top ball stop, and the top ball bounces up at a speed of nv. Prove (for example using the method of mathematical induction) that this stunt can be performed if the mass of the balls satisfies the following formula: $m_k = \frac{2m_0}{k(1+k)}$, where m_0 is the mass of the lowermost ball and $k = 1, 2, 3, \ldots, (n-1), n$. **G. 792.** In the circuit shown in the *figure* below there are identical incandescent lamps. After the switch is closed, will lamp A or B be brighter or dimmer? (Do not consider the temperature dependence of the resistances of the filament lamps.)

P. 5427. The voltage rating of each of the identical tungsten filament incandescent lamp of the circuit shown in the *figure* above is 230 V. Their current-voltage characteristics are shown in the *graph* below. The voltage supply in the circuit is 230 V. *a*) What is the resistance of an incandescent lamp at its rated voltage? *b*) What is the resistance of the filaments of lamps A and B in the open position of the switch? *c*) What is the resistance of the filaments after the switch is closed? *d*) How much power is dissipated by each incandescent lamp in the above cases? **P. 5428.** Imagine that on a day of an equinox you are lying in the sand on the beach of an equatorial country and observe the sunset. The sea is very smooth, the sky is clear blue, and at the moment when the last ray of the Sun disappears over the horizon, you suddenly stand up, so you can see the Sun's upper rim again. Estimate how long it takes for the Sun after you stand up to disappear again. **P. 5429.** An electric car accelerates uniformly from rest and reaches a speed of 108 km/h in 10 s. The radius of its wheels is 0.4 m, on the wheel there is a decorating ring of radius 0.2 m. How much time elapses from the start of the car until this narrow decorating ring will have a point which does not accelerate? What is the speed of the car at this moment? **P. 5430.** "Intensive meat production" is often cited as one of the causes of climate change. A single cow emits 160-320 litres of methane per day. There are one billion cattle in global livestock production. Estimate the thickness of the methane layer that would be formed on the surface of the Earth. (Consider the Earth as a sphere of radius 6370 km.) **P. 5431.** The rotational inertia of a spherical, uniform-density solid body of radius 10 cm with respect to a certain axis t is 10% greater than the minimum possible rotational of inertia of the sphere. How far is the axis t from the centre of the sphere? **P. 5432.** Three isolating beads having the same mass and given the same charge are stringed to a thin insulating stick fixed in a vertical position. The bottom bead is fixed and the above two beads are free to slide on the stick. At equilibrium, how many times further is the top bead from the middle bead than the middle bead from the bottom bead? **P. 5433.** The three vertical sides of a cuboid-shaped aquarium filled with water, with negligible wall thickness, reflect the light from the water. The aquarium has a width of $d = 50$ cm and a length of $L = 120$ cm. A horizontal laser beam is incident on the shorter side of the aquarium at a certain angle of incidence. The *figure* shows the top view. (The refraction index of water is $n = 4/3$.) The light beam – after being reflected several times – emerges from the aquarium parallel to the original incident light beam. At most how many reflections could occur? **P. 5434.** The electromotive force of the voltage supply shown in the *figure* increases linearly in time after switching on, from an initial value of 0 volts; $U(t) = U_0 \frac{t}{t_0}$. The switch K can be closed at any moment to connect the voltage source to the circuit. After the voltage source is turned on, how much time should elapse till the switch is closed so that the current flowing through the resistor also increases linearly in time? At what rate does the current increase then? **P. 5435.** A tube has an internal radius of R, and its axis makes an angle of α with the horizontal. The tube is rotated at a constant angular speed of ω about its axis. A small point-like body is inserted into the tube. The coefficient of kinetic friction between the wall of the tube and the small body is μ ($\mu > \operatorname{tg}\alpha$). We find that after a sufficiently long time the small body undergoes uniform straight line motion. What is the speed of the motion?