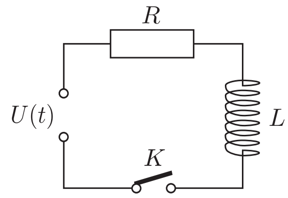
P. 5434. Az *ábrán* szereplő feszültségforrás elektromotoros ereje bekapcsolás után időben lineárisan növekszik fel a kezdeti 0 voltos értékről; $U(t)=U_0\frac{t}{t_0}$. A K kapcsoló segítségével bármelyik pillanatban rákapcsolható a feszültségforrás az áramkörre. A feszültségforrás bekapcsolása után mennyi idővel kell zárni a kapcsolót, hogy ezután az ellenálláson átfolyó áram erőssége időben lineárison nőjön? Milyen ütemben nő ekkor az áramerősség?



(*5 pont*) Közli: *Széchenyi Gábor,* Budapest

P. 5435. Egy cső belső sugara R, tengelye α szöget zár be a vízszintessel. A csövet állandó ω szögsebességgel forgatjuk a tengelye körül.

A csőbe egy pontszerűnek tekinthető, kicsiny testet helyezünk. A cső fala és a kis test közötti csúszási súrlódási együttható μ ($\mu>\operatorname{tg}\alpha$). Azt tapasztaljuk, hogy kellően hosszú idő elteltével a kis test egyenes vonalú egyenletes mozgást végez. Mekkora a mozgás sebessége?

(*6 pont*) Közli: *Balogh Péter,* Gödöllő

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Problems in Mathematics

New exercises for practice – competition K (see page 415): **K. 734.** Alex and his friends bought 6 bags of sunflower seeds and 4 bags of pumpkin seeds for 1900 HUF. Next week they bought 4 bags of sunflower seeds and 2 bags of pumpkin seeds for 1100 HUF. How much does a single bag of each type cost (assuming that the prices did not change during the week)? **K. 735.** Logic blocks were developed by Zoltán Dienes. Peter takes all the red and green disks and squares out of a set of blocks, altogether 16 pieces. No two pieces are identical, and they can be classified into two groups having the same number of elements according to each of the following properties: – either small or large, – either red or green, – either a disk or a square, – either hollow or not. Can Peter place the 16 blocks along the perimeter of a circle such that any two neighbors have exactly one of the above properties in common? **K. 736.** A company has 120 employees: plumbers, tilers, bricklayers and painters. All plumbers and bricklayers have a driving license, the others do not. The bricklayers and painters work in Pipacs street, the others in Kankalin street. The number of employees without a driving license is 64, and 84 employees work in Kankalin street. There are twice as many plumbers as painters. How many employees of each kind are there at the company? **K/C. 737.** Given two threads of known length, we can measure and mark off the sum or difference of their lengths, and also the half length of a thread by folding it into two. We are given two threads of length 2240 cm and 1760 cm. Describe a procedure to mark off a length of 10 cm by using a *single measurement* (that is, measuring the sum or difference of lengths is allowed only once, but halving a length by folding can be performed many times). **K/C. 738.** In a certain calendar, the days of a month are arranged in 7 columns. Read from left to right and then from top to bottom, each column contains the same day of the week. For a certain integer n, we select an $n\times n$ square array of days and find that their sum is 198. What is the smallest number in this square?

New exercises for practice – competition C (see page 416): **Exercises up to grade 10: K/C. 737.** See the text at Exercises **K. K/C. 738.** See the text at Exercises **K. Exercises for everyone: C. 1733.** At most how many different positive prime divisors can a 3-digit number have, if its digits are consecutive positive integers in a certain order? (Based on the idea of *Erzsébet Berkó,* Szolnok) **C. 1734.** A circle k with diameter AB has center O. We draw the circle k_1 with diameter OB, and the line parallel with AB that touches the circle k_1 at point C. This line intersects the circle k at points D_1 and D_2. Determine the angles $\angle COD_1$ and $\angle COD_2$ exactly. (Proposed by *Bálint Bíró,* Eger) **C. 1735.** Find the real solutions of the system $\sqrt{x}+\sqrt{y}=6$, $\frac{1}{x}+\frac{1}{y}=\frac{5}{16}$. (*The Mathematical Association of America*) **Exercises upwards of grade 11: C. 1736.** Let P be an interior point of side CD of a parallelogram $ABCD$, and let Q be an interior point of side AB (being parallel with CD). The line segments PA and QD intersect each other at M, while the line segments PB and QC intersect each other at N. Find a condition to have $MN\parallel AB$. (*Based on a U.S. mathematics competition problem*) **C. 1737.** Dick got two dice for his 32[th] birthday. He labelled the faces of one die with the numbers $1,2,\ldots,6$, and the faces of the other one with $0,1,2,7,8,9$. By using these dice, he can form all integers from 10 up to his age, 32, but the next number, 33, cannot be formed. Octavia uses two regular octahedra instead. Similarly, she wrote a digit on each face of both octahedra, so she can also form all integers from 10 up to her current age (in years), but not the next one. How old is Octavia now? (Proposed by *Katalin Abigél Kozma,* Győr)

New exercises – competition B (see page 418): **B. 5262.** Louisa wrote down a natural number, not containing 0 but containing at least two different digits. Then she also listed all the numbers which can be formed by permuting the digits of the original number. What is the maximum of the greatest common divisor of all the numbers (including the original one)? (*3 points*) (Proposed by *Katalin Abigél Kozma,* Győr) **B. 5263.** Prove that the sum of the squares of the medians of a triangle is not less than the square of the semiperimeter of the triangle. (*3 points*) (Proposed by *László Németh,* Fonyód) **B. 5264.** First Player and Second Player play the following game. First Player starts and prescribes arbitrarily many (even infinitely many) terms of a binary sequence (i.e., any term is 0 or 1) in a way that infinitely many terms can still be determined. Then Second Player sets the value of the first digit which has not been prescribed yet. They then repeat this procedure forever by taking turns. First Player wins if the binary sequence is periodic from a certain term, otherwise, Second Player wins. Is there a winning strategy, and if yes, who has it? (*4 points*) (Proposed by *Péter Pál Pach,* Budapest) **B. 5265.** Enlarge the incircle of a right-angled triangle by a scale factor of 2, where the center of enlargement is the vertex at the right angle. Show that this circle touches the circumcircle of the triangle. (*4 points*) (Proposed by *Viktor Vígh,* Szeged) **B. 5266.** Some football players are on holiday together. Altogether they are from k clubs and from n nations where $k<n$. Show that there are at least $n-k+1$ players having more club fellows than compatriots. (*5 points*) **B. 5267.** We are given two line segments of length p and q, and a triangle ABC determined by the

lines a, b and c (where the usual convention is used: points B and C lie on line a, and so on). Construct the point P on the circumcircle of the triangle for which the point P_a divides the line segment P_bP_c in a ratio $p:q$, where P_x is the orthogonal projection of the point P onto the line x. (*5 points*) **B. 5268.** Let I denote the incenter of the triangle ABC. Let P denote an arbitrary interior point of the triangle on the circle ABI. The reflection of the line AP about the line AI intersects the circle ABI at a point Q different from the point A. Prove that $CP=CQ$. (*6 points*) (Proposed by *Szilveszter Kocsis,* Budapest) **B. 5269.** Let $p\geqslant 19$ be an odd integer, and color the numbers $0,1,\ldots,p-1$ with two colors. For $1\leqslant i\leqslant p$, let x_i denote a random element of the set $\{0,1,\ldots,p-1\}$ (the choices are independent, and have uniform distribution). Prove that the probability of the event that $x_1,\ldots,x_p$ have the same color and p divides $x_1+\cdots+x_p$ is at least $3/(2^p p)$. (*6 points*) (Proposed by *Péter Pál Pach,* Budapest)

New problems – competition A (see page 419): **A. 833.** Some lattice points in the Cartesian coordinate system are colored red, the rest of the lattice points are colored blue. Such a coloring is called *finitely universal,* if for any finite, non-empty $A\subset\mathbb{Z}$ there exists $k\in\mathbb{Z}$ such that the point (x,k) is colored red if and only if $x\in A$. *a*) Does there exist a finitely universal coloring such that each row has finitely many lattice points colored red, each row is colored differently, and the set of lattice points colored red is connected? *b*) Does there exist a finitely universal coloring such that each row has a finite number of lattice points colored red, and both the set of lattice points colored red and the set of lattice points colored blue are connected? A set H of lattice points is called connected if, for any $x,y\in H$, there exists a path along the grid lines that passes only through lattice points in H and connects x to y. (Submitted by *Anett Kocsis,* Budapest) **A. 834.** Let $A_1A_2\ldots A_8$ be a convex cyclic octagon, and for $i=1,2\ldots,8$ let $B_i=A_iA_{i+3}\cap A_{i+1}A_{i+4}$ (indices are meant modulo 8). Prove that points $B_1,\ldots,B_8$ lie on the same conic section. **A. 835.** Let $f^{(n)}(x)$ denote the n^{th} iterate of function f, i.e $f^{(1)}(x)=f(x)$, $f^{(n+1)}(x)=f\Big(f^{(n)}(x)\Big)$. Let $p(n)$ be a given polynomial with integer coefficients, which maps the positive integers into the positive integers. Is it possible that the functional equation $f^{(n)}(n)=p(n)$ has exactly one solution f that maps the positive integers into the positive integers? (Submitted by *Dávid Matolcsi* and*Kristóf Szabó,* Budapest)

Problems in Physics

(see page 442)

M. 416. Hang a small object on the end of a door handle and gradually increase the load, with placing additional bodies on the load. Measure the angular position of the handle in equilibrium as a function of the mass hung on it. After reaching the maximum possible angle, gradually reduce the load and measure the angular position of the handle as a function of mass. Plot your data on the same graph.

G. 789. Estimate how much your body mass decreases while you sleep for eight hours peacefully! Use the following data: • During one breath 0.5 l of air is exchanged. • We breathe 15 times in a minute. • The carbon dioxide content of the exhaled air is 5 V/V%. • The exhaled air contains 6 V/V% water vapour. **G. 790.** The average consumption of a car is 6 litres/100 km. On a completely empty, winding two-lane road the car can travel 300 km. The total length of the curves on this road segment is 50 km, the radius of the curves is on average 1 km and the width of the lanes is 4 m. Estimate the reduction in fuel consumption if a careless driver makes every turn on the inside curve. **G. 791.** A juggling theoretical physicist invents the following stunt. He puts n perfectly elastic balls on top of each other with very small gaps between them. He drops the ball tower onto a hard surface, where the balls arrive at a speed of v. After a series of momentary collisions, all