and the midpoint of side AB is F. A circle k passes through the points F and C_1, and intersects the extensions of line segments A_1C_1 and B_1C_1 beyond C_1 at points P and Q, respectively. Prove that $A_1P = B_1Q$. (*4 points*) **B. 5258.** Is it true that every positive integer has a positive multiple in which the sum of the digits in decimal notation is at most 2022? (*5 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5259.** Solve the following simultaneous equations over the set of real numbers: $x^2 - 3y + 4 = z$, $y^2 - 3z + 4 = w$, $z^2 - 3w + 4 = x$, $w^2 - 3x + 4 = y$. (*4 points*) (Based on the idea of *M. Bencze,* Brassó) **B. 5260.** G and H are points of chord AB of a circle k such that $AG = GH = HB = 1$. Let F denote the midpoint of one of the arcs AB. The secants FH and FG intersect the circle again at points C and D, respectively. Show that $CD = BC^2$. (*6 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5261.** Starting Player and Second Player are playing a game on the edges of a complete graph of 100 vertices. They take turns in colouring an edge of the graph that has not been coloured before. In each step, Starting Player colours his edge red, and Second Player colours his edge blue. The game terminates and Starting Player wins if there is a set of four vertices such that all the six connecting edges are red. The game terminates and Second Player wins if there is a set of four vertices such that all the six connecting edges are blue. The game terminates with a draw if there is no such set of four vertices but there remain no further vertices to colour. Who has a winning strategy? (*6 points*)

New problems – competition A (see page 354): **A. 830.** For $H \subset \mathbb{Z}$ and $n \in \mathbb{Z}$, let h_n denote the number of finite subsets of H in which the sum of the elements is n. Does there exist $H \subset \mathbb{Z}$, for which $0 \notin H$, and h_n is a (finite) even number for every $n \in \mathbb{Z}$? (The sum of the elements of the empty set is 0.) (Submitted by *Csongor Beke,* Cambridge) **A. 831.** In triangle ABC let F denote the midpoint of side BC. Let the circle passing through point A and tangent to side BC at point F intersect sides AB and AC at points M and N, respectively. Let line segments CM and BN intersect in point X. Let P be the second point of intersection of the circumcircles of triangles BMX and CNX. Prove that points A, F and P are collinear. **A. 832.** Let us assume that the number of offsprings for every man can be $0, 1, \ldots$ or n with probabilities $p_0, p_1, \ldots, p_n$ independently from each other, where $p_0 + p_1 + \cdots + p_n = 1$ and $p_n \neq 0$. (This is the so called Galton–Watson process.) Which positive integer n and probabilities $p_0, p_1, \ldots, p_n$ will maximize the probability that the offsprings of a given man go extinct in exactly the tenth generation?

Problems in Physics

(see page 378)

M. 415. Taking advantage of the warm summer weather, measure how the horizontal range of a jet of water launched from a hose at the ground depends on the water flow and the angular position of the nozzle.

G. 785. In cloudy weather, it either rains or it doesn't. What determines whether the raindrops (or the ice crystals) in a cloud fall off due to gravity or stay in the cloud? **G. 786.** One day in December and one in June, in Ecuador, at noon, with solar eclipse glasses on, we face to the Sun. What do we see, which way does the Sun move in the sky, to the right or to the left? **G. 787.** Research the internet and find out in the case of water between the temperature values of 0 °C and 100 °C the largest percentage difference of the following quantities: density, speed of sound, surface tension, and specific heat. To what temperature values (in Celsius degree) do the maximum and minimum values of these quantities belong? (Always relate the difference in percent to the maximum value.) Also indicate the sources of your data. **G. 788.** A boy takes a boat across a river to the

pier directly opposite, then immediately turns around and rows back to the starting point. The river is 288 m wide, the water flows at a speed of 1 m/s, and the speed of the boat relative to the water is 2.6 m/s. The boy also tries that he rows upstream 288 m and then rows back to the starting point. Calculate the times for the two movements of the boat.

P. 5418. Two balls, kicked at different angles but with the same initial speed, land at the same distance. The ball with the higher trajectory flew twice as long as the other. What is the relationship between the peak heights of the two trajectories? At what angles were the balls kicked? **P. 5419.** The magnitude of the acceleration of a point-like body moving at a constant speed of 6 m/s in a horizontal plane is constant. The length of the path of the body between points A and B is 1.2 times the magnitude of the displacement vector. It takes 2 seconds for the body to cover this path. What is its acceleration? **P. 5420.** *a*) At what angle of α is the system in the *figure* in equilibrium if there is no friction on the slope? *b*) For what angles of α are the objects in equilibrium if the coefficient of friction on the slope is $\mu = 0.2$? *c*) What is the acceleration and the direction of motion of the objects if $\alpha = 35^\circ$ and the coefficient of friction is $\mu = 0.15$? What is the ratio of the two tensions in this case? **P. 5421.** At the bottom of the sea, 150 m below the surface, lies a sunken, mainly steel ship, which once had a 1000 tons displacement. Divers would like to bring this ship the surface. To do this, the divers create arched sections in the ship, into which the atmospheric air from the surface is pumped through long tubes by means of compressors. At least how much work do the compressors have to do in order to raise the ship off the seabed? We can assume that both the sea and the air has a temperature of 15 °C. **P. 5422.** A closed, cylindrical tank of length $L = 40$ cm, is made of heat-conducting walls and is divided into two parts with a thin piston. There is some ideal gas in both parts of the tank. Initially, the axis of the tank is vertical and the piston is in equilibrium, exactly at the middle of the tank. Then the tank is slowly turned such that its symmetry axis turns 90°, which causes the piston to move a distance of 10 cm. How much would the piston have moved if the tank had been rotated by 180° instead of 90°? The temperature is constant all the time. **P. 5423.** A rectangular frame of wire is placed next to a long, straight conducting wire such that its plane is perpendicular to the wire, as it is shown in the *figure.* The midpoint of one of the edges of the frame is the closest to the wire, and it is at a distance of d from the wire. Do the two wires attract or repel each other if the straight wire carries a current of magnitude I_1 and the conducting frame carries a current of magnitude I_2? **P. 5424.** The capacitor system shown in the *figure* is made of 5 square-shaped uncharged metal plates of area A. The distance between the plates is ℓ or 2ℓ, and the edge effects are negligible because $\ell^2 \ll A$. Between the plates, in the white regions there is air and in the brown regions there is some insulating material of relative dielectric constant ε_r. In both condensers which contain dielectric, the dielectric material fills half of the area between the plates of the condensers. What is the equivalent capacitance of the arrangement? **P. 5425.** A ray of light passes from air into water of refractive index $n = 1.33$. What is the angle of incidence if during refraction the component of the speed of light ray which is perpendicular to the boundary does not change? **P. 5426.** A photon rocket is an imaginary rocket whose engine converts fuel into photons, which are then ejected into one direction parallel to each other. During a long-duration space mission, the rocket, starting from rest and moving in a straight path, accelerates to some speed, then with its engine running in the opposite direction, it brakes to a stop at the end of its journey. During this time, the mass of the rocket is reduced to one-quarter of its original value. What was the maximum speed of the rocket?