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Problems in Mathematics

New exercises for practice – competition K (see page 351): **K. 729.** What angle is enclosed by the hands of the tower hall clock 2022 minutes before it strikes midnight? (Based on the idea of *K.A. Kozma,* Győr) **K. 730.** Eight chords are drawn in a circle, such that they have the largest possible number of intersections. Into how many regions will the eight chords divide the disc bounded by the circle? **K. 731.** A 4×6 rectangle is to be covered, without overlaps, with tiles congruent to the L-shape shown in the *figure*. The L-shaped tiles may be rotated or turned over as needed. Are there at least 36 different arrangements possible? **K/C. 732.** Four mathematics teachers are all younger than 70 years, and the age of each of them is a prime number of years. How old is the youngest if their average age is 60 and all their ages are different? **K/C. 733.** What is the area of the smallest rectangle that has an inscribed parallelogram in which one angle is 60°, sides are 4 cm and 6 cm long, and two sides lie on two sides of the rectangle?

New exercises for practice – competition C (see page 352): **Exercises up to grade 10: K/C. 732.** See the text at Exercises **K. K/C. 733.** See the text at Exercises **K. Exercises for everyone: C. 1728.** Find the exact solutions of the equation $-\frac{1}{6}x+\frac{1}{2}=\{x\}$. ($\{x\}$ denotes the fractional part of x, that is, the difference between x and the greatest integer not greater than x.) **C. 1729.** Semicircles k_1 and k_2, respectively, are drawn outside a square $ABCD$, over the sides BC és CD as diameters. The midpoints of the semicircular arcs are E and F, and the midpoints of line segments DE and AF are P and Q, respectively. Show that P lies on diagonal AC and Q lies on diagonal BD of the square. **C. 1730.** Find all decimal numbers of the form $\overline{0.abc}$ where a, b, c are digits, $a\neq 0$, and $\overline{0.abc}=\frac{a}{a+b+c}$. (*Croatian problem*) **Exercises upwards of grade 11: C. 1731.** The parallel sides of a trapezium $ABCD$ are $AB>CD$. The midline of the trapezium intersects diagonal AC at E and diagonal BD at F. The length of line segment CD is the *a*) arithmetic *b*) geometric mean of line segments AB and EF. In which of the two cases will the ratio $\frac{AB}{CD}$ have a larger value? **C. 1732.** Let U denote the set of prime numbers greater than 337 but not greater than 733. How many 4-element subset does U have that contain 467 or 499 as an element?

New exercises – competition B (see page 353): **B. 5254.** Prove that the difference of the squares of any two odd numbers not divisible by 3 is divisible by 24. (*3 points*) (*Journal of Mathematics and Science Didactics,* 1943) **B. 5255.** Vertex A of a triangle ABC is reflected about vertex B, B is reflected about C, and C is reflected about A. The reflections are points C_1, A_1 and B_1, respectively. Show that there exists a triangle with sides of lengths AA_1, BB_1 and CC_1. (*3 points*) **B. 5256.** In the lottery game, five numbers are drawn out of ninety every week. Andrew fills out a single lottery ticket with the same five numbers in each of the 52 weeks of the year. Belle uses a different scheme. She plays only once a year with 52 tickets simultaneously: she fills them out in pairwise different ways. Is it true that both of them have the same chance of having a ticket with five correct numbers? (*4 points*) **B. 5257.** In an acute-angled triangle ABC, the heights are AA_1, BB_1, CC_1, and the midpoint of side AB is F. A circle k passes through the points F and C_1, and intersects the extensions of line segments A_1C_1 and B_1C_1 beyond C_1 at points P and Q, respectively. Prove that $A_1P = B_1Q$. (*4 points*) **B. 5258.** Is it true that every positive integer has a positive multiple in which the sum of the digits in decimal notation is at most 2022? (*5 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5259.** Solve the following simultaneous equations over the set of real numbers: $x^2 - 3y + 4 = z$, $y^2 - 3z + 4 = w$, $z^2 - 3w + 4 = x$, $w^2 - 3x + 4 = y$. (*4 points*) (Based on the idea of *M. Bencze,* Brassó) **B. 5260.** G and H are points of chord AB of a circle k such that $AG = GH = HB = 1$. Let F denote the midpoint of one of the arcs AB. The secants FH and FG intersect the circle again at points C and D, respectively. Show that $CD = BC^2$. (*6 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5261.** Starting Player and Second Player are playing a game on the edges of a complete graph of 100 vertices. They take turns in colouring an edge of the graph that has not been coloured before. In each step, Starting Player colours his edge red, and Second Player colours his edge blue. The game terminates and Starting Player wins if there is a set of four vertices such that all the six connecting edges are red. The game terminates and Second Player wins if there is a set of four vertices such that all the six connecting edges are blue. The game terminates with a draw if there is no such set of four vertices but there remain no further vertices to colour. Who has a winning strategy? (*6 points*)

New problems – competition A (see page 354): **A. 830.** For $H \subset \mathbb{Z}$ and $n \in \mathbb{Z}$, let h_n denote the number of finite subsets of H in which the sum of the elements is n. Does there exist $H \subset \mathbb{Z}$, for which $0 \notin H$, and h_n is a (finite) even number for every $n \in \mathbb{Z}$? (The sum of the elements of the empty set is 0.) (Submitted by *Csongor Beke,* Cambridge) **A. 831.** In triangle ABC let F denote the midpoint of side BC. Let the circle passing through point A and tangent to side BC at point F intersect sides AB and AC at points M and N, respectively. Let line segments CM and BN intersect in point X. Let P be the second point of intersection of the circumcircles of triangles BMX and CNX. Prove that points A, F and P are collinear. **A. 832.** Let us assume that the number of offsprings for every man can be $0, 1, \dots$ or n with probabilities $p_0, p_1, \dots, p_n$ independently from each other, where $p_0 + p_1 + \dots + p_n = 1$ and $p_n \neq 0$. (This is the so called Galton–Watson process.) Which positive integer n and probabilities $p_0, p_1, \dots, p_n$ will maximize the probability that the offsprings of a given man go extinct in exactly the tenth generation?

Problems in Physics

(see page 378)

M. 415. Taking advantage of the warm summer weather, measure how the horizontal range of a jet of water launched from a hose at the ground depends on the water flow and the angular position of the nozzle.

G. 785. In cloudy weather, it either rains or it doesn't. What determines whether the raindrops (or the ice crystals) in a cloud fall off due to gravity or stay in the cloud? **G. 786.** One day in December and one in June, in Ecuador, at noon, with solar eclipse glasses on, we face to the Sun. What do we see, which way does the Sun move in the sky, to the right or to the left? **G. 787.** Research the internet and find out in the case of water between the temperature values of 0 °C and 100 °C the largest percentage difference of the following quantities: density, speed of sound, surface tension, and specific heat. To what temperature values (in Celsius degree) do the maximum and minimum values of these quantities belong? (Always relate the difference in percent to the maximum value.) Also indicate the sources of your data. **G. 788.** A boy takes a boat across a river to the