is even, then the k-element subsets of an n-element set S can be paired up so that the symmetric difference of every pair should have exactly 2 elements? (*6 points*)

New problems – competition A (see page 291): **A. 827.** Let $n > 1$ be a given integer. In a deck of cards the cards are of n different suites and n different values, and for each pair of a suite and a value there is exactly one such card. We shuffle the deck and distribute the cards among n players giving each player n cards. The players' goal is to choose a way to sit down around a round table so that they will be able to do the following: the first player puts down an arbitrary card, and then each consecutive player puts down a card that has a different suite and different value compared to the previous card that was put down on the table. For which n is it possible that the cards were distributed in such a way that the players cannot achieve their goal? (The players work together, and they can see each other's cards.) (Proposed by *Anett Kocsis,* Budapest) **A. 828.** Triangle ABC has incenter I and excircles Ω_A, Ω_B, and Ω_C. Let ℓ_A be the line through the feet of the tangents from I to Ω_A, and define lines ℓ_B and ℓ_C similarly. Prove that the orthocenter of the triangle formed by lines ℓ_A, ℓ_B, and ℓ_C coincides with the Nagel point of triangle ABC. (The Nagel point of triangle ABC is the intersection of segments AT_A, BT_B, and CT_C, where T_A is the tangency point of Ω_A with side BC, and points T_B and T_C are defined similarly.) (Proposed by *Nikolai Beluhov,* Bulgaria) **A. 829.** Let G be a simple graph on n vertices with at least one edge, and let us consider those $S : V(G) \to \mathbb{R}^{\geqslant 0}$ weighings of the vertices of the graph for which $\sum_{v\in V(G)} S(v) = 1$. Furthermore define $f(G) = \max_S \min_{(v,w)\in E(G)} S(v)S(w)$, where S runs through all possible weighings. Prove that $f(G) = \frac{1}{n^2}$ if and only if the vertices of G can be covered with a disjoint union of edges and odd cycles. ($V(G)$ denotes the vertices of graph G, $E(G)$ denotes the edges of graph G.)

Problems in Physics

(see page 313)

M. 414. Measure the coefficient of kinetic friction between several sheets of sandpaper with different grit sizes and a wooden block.

G. 781. Boil water in a large pot on the stove. Put some cool water in a thin-walled glass, then immerse the glass of water into the boiling water so that it does not touch the walls of the pot. Will the water in the glass boil if we wait for a long enough time? **G. 782.** A bicycle is moving uniformly along a horizontal path at a speed of 3 m/s. Its wheels have a diameter of 70 cm. Choose an arbitrary point on the circumference of the wheel and at different positions of the wheel draw the velocity vectors and the acceleration vectors of this point starting from one common point for each quantity, that is draw the velocity and acceleration hodographs. **G. 783.** There is a point-like light source at the centre of a uniform glass ball of radius R and of refractive index n. The sphere is observed from the outside. Where do we see the image of the light source? **G. 784.** The *figure* shows a whole range of simple machines. Friction and the masses of pulleys and levels are negligible. Into which direction will the lowermost object start moving?

P. 5409. The *figure* shows a whole range of simple machines. Friction and the masses of pulleys and levels are negligible. What are the values of the tension in the threads? **P. 5410.** The peregrine falcon can travel long distances without flapping its wing. Doing so, its movement has two parts. In the first part, it circles with its wings extended and rises in an upward flowing column of warm air (thermals) at a vertical speed of v_1. In the second part, it leaves the thermal at an angle of α with respect to the horizontal and glides at a constant speed to the next thermal at a distance of L. The glide speed v_2 is approximately directly proportional to the sine of the angle α (the direction of glide with the horizontal): $v_2 = k\sin\alpha$, where k is a known constant. *a*) To what minimum height must a peregrine rise in the thermal so that the time of its rising and gliding motion should be the shortest possible? *b*) At least how much time is needed for the peregrine to move from the bottom of a thermal to the bottom of the next thermal? *c*) Determine the glide angle which belongs to the motion with the optimal flight time. *Data:* $v_1 = 2\ \frac{\mathrm{m}}{\mathrm{s}}$, $k = 10\ \frac{\mathrm{m}}{\mathrm{s}}$, $L = 2$ km. **P. 5411.** A satellite orbits the Earth in an elliptical orbit of numerical eccentricity $c/a = e$, with a period of T. How long does it take for the satellite to go from point A to point B, shown in the *figure?* **P. 5412.** If a gas is cooled (at constant pressure), then at a sufficiently low temperature the gas will usually liquefy (condense). However, this only happens over a certain pressure range. The *figure* shows the "phase diagram" of carbon dioxide. What are the values of the minimum and the maximum pressure at which this condensation can occur as described above? What happens if cooling is carried out at pressures higher or lower than this range? (Translation of the labels in the figure is as follows: légnemű = gas, folyadék = liquid, szilárd = solid, hármaspont = triple point, kritikus pont = critical point, szuperkritikus állapot = supercritical fluid.) **P. 5413.** A converging lens with a focal length of 20 cm is placed on a convex spherical mirror as shown in the *figure.* What should the radius of curvature of the mirror be in order that a vertical parallel beam of light incident on the lens remain parallel after reflection from the system? **P. 5414.** We formed a circle of radius R from a piece of metal wire and from the same wire we made one of the diameters of the circle as well. What should the length of the arcs $AB = AC$ be in order that the equivalent resistance between points A and B be the same as the equivalent resistance between points B and C? **P. 5415.** From a piece of wire of negligible resistance a V shaped figure was bent. The wire has no insulation, the angle between the two parts of the V is $\alpha = 45^\circ$. It is placed horizontally into a magnetic field whose induction $\boldsymbol{B}$ is perpendicular to the plane of the wire. The magnitude of this induction $\boldsymbol{B}$ changes with time according to $B(t) = B_0/t_0 \cdot t$, where B_0 and t_0 are known constants. A metal rod, also without insulation, is placed onto the V shaped wire, initially it is fixed, as shown in the *figure.* The resistance of a unit length of the rod is r. *a*) How much heat is produced in the metal rod in a time of t_0? *b*) At time t_0 from the moment of switching on (time $t = 0$), the change in magnetic induction ceases. At this instant we begin to move the metal rod (which was fixed till this time) in the horizontal plane and perpendicularly to the rod at a constant speed of v_0. What should this speed be in order that the value of the current in the rod should not change? *c*) By what factor will the heat produced in the moving rod be greater than that produced in a static rod, if the rod is moved for a time of $2t_0$? **P. 5416.** There are five electrons in a region of length 1.1 nm with respect to which the width and thickness of the region is negligibly small. The potential energy in this region is zero, and outside it is very big. We can neglect the interaction of the electrons with each other. *a*) What is the minimum energy required to excite the electrons in the system? *b*) What is the wavelength of the electromagnetic wave that can produce this excitation? Where is this electromagnetic wave in the spectrum? **P. 5417.** A small body of mass m is placed (but not fixed) onto the top of a thin cylindrical ring of radius R and of negligible mass being at rest on the horizontal ground. The system is displaced from its unstable equilibrium position. As the ring rolls faster and faster, the small body flies off somewhere. *a*) What is the least value of the coefficient of static friction between the surfaces in contact if neither the small body on the ring nor the ring on the ground is skidding during the motion? *b*) Where will the small object hit the ground?