

New exercises – competition B (see page 227): **B. 5238.** Solve the following equation over the set of positive integers: $(k+n)! = k^3 + n^3 + (k+n)(3kn-1)$. (3 points) (Proposed by *M. Szalai*, Szeged) **B. 5239.** The sides a , b and c of a triangle, in this order, form an arithmetic sequence. Show that the centre of the inscribed circle divides the angle bisector drawn to side b in a $1 : 2$ ratio. (3 points) **B. 5240.** Show that every positive integer n has a multiple in which the sum of the digits is n . (4 points) (Proposed by *Cs. Sándor*, Budapest) **B. 5241.** In a triangle ABC , the centre of the circumscribed circle is O , and $\angle ABC > 90^\circ$. The tangent drawn to the circumscribed circle at C intersects line AB at point P , and the perpendicular drawn from P to BC intersects line OC at Q . Prove that AB is perpendicular to AQ . (4 points) (Proposed by *Z. L. Nagy*, Budapest) **B. 5242.** Let m and n denote arbitrary positive integers. Consider those lattice points $(x; y)$ in the Cartesian coordinate plane for which $1 \leq x \leq m$ and $1 \leq y \leq n$. What is the maximum possible number of points that can be selected out of these mn lattice points such that no four points selected should form a non-degenerate parallelogram? (6 points) (Proposed by *E. Füredi*, Budapest) **B. 5243.** In a triangle ABC , $\angle CAB = 48^\circ$ and $\angle ABC = 54^\circ$. D is an interior point of the triangle such that $\angle CDB = 132^\circ$ and $\angle BCD = 30^\circ$. Prove that the line segments forming the polygon $ACDB$ cannot form a triangle. (5 points) **B. 5244.** Determine all integers $n > 4$ such that $(k, n) > 1$ for all composite numbers k less than n . (5 points) (Proposed by *S. Róka*, Nyíregyháza) **B. 5245.** a) Prove that there exist infinitely many, pairwise non-similar triangles in which the lengths of the sides are integers, and one angle is 3 times as large as another. b) Is there a triangle with the above property in which the lengths of the sides are all at most 10? (6 points) (Based on the idea of *M. Hujter*, Budapest) **April puzzle***. Place on a chessboard six white chess pieces selected from two chess sets such that however a black piece is placed on a vacant field, it can be eliminated immediately.

New problems – competition A (see page 229): **A. 824.** An infinite set S of positive numbers is called thick, if in every interval of the form $[1/(n+1), 1/n]$ (where n is an arbitrary positive integer) there is a number which is the difference of two elements from S . Does there exist a thick set such that the sum of its elements is finite? (Proposed by *Gábor Szűcs*, Szikszó) **A. 825.** Find all functions $f : \mathbb{Z}^+ \rightarrow \mathbb{R}^+$ that satisfy $f(nk^2) = f(n)f^2(k)$ for all positive integers n and k , furthermore $\lim_{n \rightarrow \infty} \frac{f(n+1)}{f(n)} = 1$. **A. 826.** An antelope is a chess piece which moves similarly to the knight: two cells (x_1, y_1) and (x_2, y_2) are joined by an antelope move if and only if $\{|x_1 - x_2|, |y_1 - y_2|\} = \{3, 4\}$. The numbers from 1 to 10^{12} are placed in the cells of a $10^6 \times 10^6$ grid. Let D be the set of all absolute differences of the form $|a - b|$, where a and b are joined by an antelope move in the arrangement. How many arrangements are there such that D contains exactly four elements? (Proposed by *Nikolai Beluhov*, Bulgaria)

Problems in Physics

(see page 251)

M. 413. Measure the refractive index of cooking oil.

G. 777. We would like to heat room temperature espresso coffee by “steaming”. Estimate how much the “quality” of the coffee deteriorates, that is, how much the coffee concentration decreases. **G. 778.** The figure shows the wet track of a bicycle on a dry asphalt after the bicycle passed a puddle. Did the bike move from the left to the right or

* Out of competition. A possible solution will be shown on the cover of the May issue.

from the right to the left? Which trail was made by the front wheel and which one was made by the rear one? **G. 779.** Would there be tides on the Moon if its surface was covered by oceans and landmasses? **G. 780.** Are fish disturbed by fishermen talking 2 metres away from the shore? (The speed of sound is 340 m/s in air and 1500 m/s in water.)

P. 5400. One of the little prince's spherical planets is spinning so fast that the gravitational acceleration on its equator is zero. In which direction do the trees on the planet grow? **P. 5401.** A small (considered point-like) but heavy body is held by two ropes having the same length and approximately the same working load limit. The upper ends of the ropes are slowly moved apart along a horizontal line. When the angle between the ropes becomes 2α , one of the ropes breaks and the body begins to swing as a pendulum about the fixed end of the other rope. What could the value of α be if the other rope does not break during the motion? **P. 5402.** A point-like weight m is attached to a thin ring of radius R and of negligible mass. The ring moves out of its unstable equilibrium position shown in the *figure*, and it starts slipping on the ground when the displacement of its centre is just R . What is the coefficient of friction between the ring and the horizontal ground? **P. 5403.** Someone suggested building a mobile dam from iron sheets: a horizontal one and another which is attached to it and is tilted at an angle of 45° . The length of the edges of the sheets which are perpendicular to the common intersection line of the two sheets are 1 metre and $\sqrt{2}$ metres, and the width of the iron sheets is 2 cm. "This structure is pressed against the ground not only by the weight of the iron plates but also by the pressure of the water" – argued the inventor. *a)* What is the least value of the coefficient of static friction between the mobile dam and the ground in order that the dam should be able to protect safely even if the water level is maximum (that is: the level of the water reaches the top edge of the slant iron sheet)? *b)* Where is the point of application of the vertical normal force exerted by the ground on the mobile barrier of this shape, if we use iron sheets having a thickness less than 2 cm or more than 2 cm? Can the mobile dam overturn in the case of the highest water level (i.e. when the water just to the top of the tilted iron plate)? Investigate the two options for installing the mobile dam: *(i)* the tilted surface slopes towards the water; *(ii)* the tilted surface slopes towards the area to be protected (see the *figure*). **P. 5404.** An ideal Carnot heat engine, with the help of heat reservoirs of temperatures T_1 and T_2 ($T_2 < T_1$), can perform W work in each cycle (through isothermal and adiabatic processes). How will the efficiency of the heat engine change if the small friction between the piston in the working cylinder causes that $2q$ heat in each cycle is released ($q \ll W$), and this heat is absorbed evenly by the two heat reservoirs? **P. 5405.** The sum of the currents through two resistors is I . Prove that the total dissipated power in the two resistors is minimum if the voltages across the two resistors are equal. **P. 5406.** What is the maximum potential difference that can be created with the help of two alike capacitors and a battery of electromotive force U ? The capacitors can be rearranged and connected into a circuit again after they were charged. **P. 5407.** Protons initially considered stationary in a linear accelerator at CERN are accelerated along a path of $L = 30.0$ m through a voltage of $U = 500$ MV. The electric field in the accelerator can be considered uniform. How long does it take for the protons to travel the distance L ? **P. 5408.** A simple pendulum is attached to the top of a rod mounted to a trolley of mass $M = 3m$. The trolley is on a horizontal surface. The length of the thread of the pendulum is $L = 50$ cm, and the mass of the point-like bob at its end is $m = 0.15$ kg. Initially the objects are at rest. Then the pendulum bob is displaced such that its thread is tight and horizontal, and then released without initial speed. Friction is negligible everywhere. *a)* What is the speed of the trolley, when the angle between the thread and the vertical is $\alpha = 60^\circ$? *b)* What is the tension in the thread at this position?