numbers greater than 3 that occur in the Fibonacci sequence are of the form $4k+1$. (*5 points*) **B. 5236.** Let $a$, $b$, $c$ denote positive real numbers such that $abc=1$. Show that $a+a^2+a^3+b+b^2+b^3+c+c^2+c^3 \leqslant (a^2+b^2+c^2)^2$. (*6 points*) (Proposed by *M. Lovas,* Budapest and *M. Rozenberg,* Israel) **B. 5237.** In a triangle, $r$ denotes the inradius, $R$ is the circumradius, and $s$ denotes the semiperimeter. Prove that if $r+2R=s$ then the triangle is right angled. (*6 points*) (Proposed by *R. Fridrik,* Szeged)

**New problems – competition A** (see page 159): **A. 821.** *a*) Is it possible to find a function $f:\mathbb{N}^2\to\mathbb{N}$ such that for every function $g:\mathbb{N}\to\mathbb{N}$ and positive integer $m$ there exists $n\in\mathbb{N}$ such that set $\big\{k\in\mathbb{N}: f(n,k)=g(k)\big\}$ has at least $m$ elements? *b*) Is it possible to find a function $f:\mathbb{N}^2\to\mathbb{N}$ such that for every function $g:\mathbb{N}\to\mathbb{N}$ there exists $n\in\mathbb{N}$ such that set $\big\{k\in\mathbb{N}:\ f(n,k)=g(k)\big\}$ has an infinite number of elements? **A. 822.** Is it possible to find rational numbers $p$, $q$ and $r$ such that $p+q+r=0$ and $pqr=1$? (Proposed by *Máté Weisz,* Cambridge) **A. 823.** For positive integers $n$ consider the lattice points $S_n=\big\{(x,y,z):\ 1\leqslant x\leqslant n,\ 1\leqslant y\leqslant n,\ 1\leqslant z\leqslant n,\ x,y,z\in\mathbb{N}\big\}$. Is it possible to find a positive integer $n$ for which it is possible to choose more than $n\sqrt{n}$ lattice points from $S_n$ such that for any two chosen lattice points at least two of the coordinates of one is strictly greater than the corresponding coordinates of the other? (Proposed by *Endre Csóka,* Budapest)

## Problems in Physics

(see page 186)

**M. 412.** Place several muffin cupcake paper cases into each other and carry out drop experiments. Measure how the terminal speed of the cases depends on the number of cases. Determine the drag coefficient of the cases.

**G. 773.** The Earth–Moon system revolves about the centre of mass of the two celestial bodies with a period of 27.32 days, relative to the distant fixed stars. However, in comparison, more than two days more time elapses between two full Moons, namely an average of 29.53 days. Explain the difference between the two period values, and with a simplified calculation show that the difference between them is indeed about two days.

**G. 774.** The *diagram* on page 187 shows the surface flow velocity profile for the river Danube at bridge Erzsébet on 10 Marc 2018. On the horizontal axis the distance $s$ in metres measured from the left riverbank, and on the vertical axis the speed of the water $v$ in m/s can be seen. The attached *table* shows the recorded data. Estimate the distance at which the river would drift our boat down, if we were to row at a constant velocity of 1 m/s, perpendicularly to the riverbank, to a ship which is at a distance of 221 metres from the left riverbank. **G. 775.** In a thermally insulated flask of negligible heat capacity, there is 1 kg very cold crushed ice, to which 1 kg hot water of temperature 100 °C is poured. What was the temperature of the ice originally if finally there are 2 litres of water at a temperature of 0 °C in the flask? **G. 776.** Watch glasses are to be sterilized in a research laboratory by using UV light. In the sterilization process UV light of total energy of 150 mJ should fall onto a 1 cm$^2$ surface area of the watch glass. Estimate how long should the UV lamp be operated for this, if

| $s$ [m] | $v$ [m/s] | $s$ [m] | $v$ [m/s] |
|---|---|---|---|
| 0 | 0.00 | 119 | 1.21 |
| 17 | 0.41 | 136 | 1.14 |
| 34 | 1.00 | 153 | 1.17 |
| 51 | 1.05 | 170 | 1.17 |
| 68 | 1.15 | 187 | 1.10 |
| 85 | 1.19 | 204 | 1.07 |
| 102 | 1.26 | 221 | 1.02 |

the lamp is point-like and it is 75 cm above the watch glass. According to the factory data, the intensity of the UV light at a distance of 1 m from the lamp is 125 $\mu$W/cm$^2$.

**P. 5391.** A stone is dropped into a deep well. The sound of the splash is heard 4.25 s after the drop. How deep is the well if we carry out the calculation with $g = 10$ m/s$^2$ and $v_{\text{sound}} = 320$ m/s? What is the depth of the well if instead we use the values of $g = 9.81$ m/s$^2$ and $v_{\text{sound}} = 340$ m/s? (We neglect air resistance.) **P. 5392.** A thin water jet ejected vertically out of the central nozzle of a fountain reaches a height of $H$. The "flow rate" of the water jet, i.e. the volume of water flowing out per unit time is $\Phi = \frac{\Delta V}{\Delta t}$. At what height $h$ does a ball of mass $m$ float when it is placed into the water jet? (We can assume that the total cross-section of the water jet reaches the ball, and that the water splashes off from the ball in horizontal direction.) **P. 5393.** Threads are attached to two small balls of masses $m$ and $M = 3m$ such that the other ends of the threads are fixed at the same height, as it is shown in the *figure on the left.* The centres of the balls are then at a depth of $L$ below the suspension. Then the ball with the smaller mass is raised such that the thread attached to it becomes horizontal (*right figure*), and then this ball is released. The two balls collide head-on and totally elastically. *a*) Right before the collision what is the total force exerted on the suspension by the two threads? *b*) What is this total force right after the collision? *c*) Between the first and the second collisions of the balls what is the greatest angle enclosed by the two threads? *d*) In the case of *c*) what is the direction and the magnitude of the total force exerted by the threads on the suspension? *e*) What is the angle enclosed by the threads and the vertical when the second collision occurs? **P. 5394.** The semi-major and semi-minor axes of an ellipse-shaped uniform-density plate of mass $m$ are $a$ and $b$, respectively. What is the rotational inertia of the plate with respect to an axis which is perpendicular to the plane of the plate and goes through one end of the major axis of length $2a$? (The problem can also be solved by elementary considerations not requiring higher mathematics.) **P. 5395.** One year ago, in March 2021, the first voice message arrived from the Mars Perseverance Rover (`go.nasa.gov/3ly2OE4`). What might the speed of sound in the atmosphere of the Mars be? **P. 5396.** A sample of diatomic ideal gas at a temperature $T_0$ is enclosed in a vertical, thermally insulated container by an easily moveable and thermally insulated piston. The gas is slowly heated, thus its volume begins to increase. During heating, when the volume of the gas has just doubled, the piston in the cylinder got stuck in the rim which narrowed the cylinder. Determine the final temperature of the gas $T$, if it is known that 80% of the heat added to the gas was used to increase the internal energy. **P. 5397.** A small object of charge $Q = 10^{-9}$ C is fixed on an insulated stand, which is at a distance of $d = 10$ cm from a big grounded metal plate. *a*) What is the surface charge density of the metal plate at its point $P$, which is the closest to the small object? *b*) What is the distance between $P$ and that point of the plate at which the surface charge density of the plate is one-third of the maximum surface charge density value? **P. 5398.** A digital camera has an objective lens of focal length of 35 mm, and its near point is at a distance of 25 cm. The near point is that smallest distance from the sensor from which the objective can still focus. *a*) How does the distance of the near point change if a ring is placed between the lens and the camera, such that the lens gets 12 mm further away from the sensor? *b*) Take a picture of an object at the near point, with and without using the ring. What is the ratio of the sizes of the two images? **P. 5399.** From a thin metal plate of width $\delta$ a large conical surface was welded. A current of $I$ flows from the vertex of the cone $A$ to a point $B$ which lies on a slant height of the cone. Determine both the magnitude and the direction of the current density vector at point $C$ on the surface of the cone, if $C$ opposite to $B$. It is known that the distance of $AB$ is equal to $3R$ and the distance between points $B$ and $C$ is $2R$.