P. 5397. Egy $Q=10^{-9}$ C töltésű kicsiny testet egy nagy méretű, földelt fémlemeztől $d=10$ cm távolságban szigetelő állványon rögzítettünk.

a) Mekkora a fémlemez felületi töltéssűrűsége a kicsiny testhez legközelebb eső P pontjában?

b) Milyen messze van P-től az a pont, ahol a fémlemez felületi töltéssűrűsége a maximális értéknek egyharmada?

(*4 pont*) Közli: *Holics László,* Budapest

P. 5398. Digitális fényképezőgépen 35 mm gyújtótávolságú objektív található, melynek közelpontja 25 cm. A közelpont az a szenzortól mért legkisebb távolság, ahonnan az objektív még képes fókuszálni.

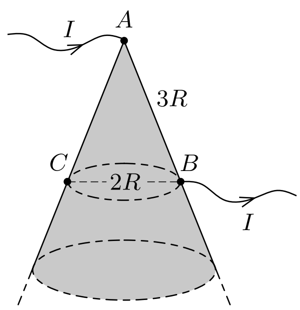
a) Hogyan változik meg a közelpont távolsága, ha az objektív és a fényképezőgép közé egy közgyűrűt helyezünk, melynek hatására az objektív 12 mm-rel messzebbre kerül a szenzortól?

b) Készítsünk egy közelpontba helyezett tárgyról felvételt közgyűrűvel és anélkül. Hogyan aránylik egymáshoz ezen két kép nagysága?

(*5 pont*) Közli: *Széchenyi Gábor,* Budapest

P. 5399. Egy vékony, δ vastagságú fémlemezből nagy, kúp alakú felületet hegesztettünk össze. A kúp A-val jelölt csúcsába I erősségű áramot vezetünk, majd az egyik alkotón lévő B pontból elvezetjük azt. Határozzuk meg a B-vel átellenes C pontban az áramsűrűség-vektor irányát és nagyságát! Ismert, hogy az AB távolság értéke $3R$, míg a B és C pontok távolsága $2R$.

(*6 pont*) Közli: *Vigh Máté,* Biatorbány



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Problems in Mathematics

New exercises for practice – competition K (see page 155): **K. 724.** Julie cut a pizza into identical slices. Then she ate a few slices, but 3 slices remained. With a little calculation, she observed that she had eaten 3/4 of the whole pizza, plus 3/4 of a slice. How many slices were there? **K. 725.** We filled in the nine fields of a 3×3 table one by one, according to the following rule: in each field, we entered the number of adjacent fields (i.e. fields sharing a common side with it) that had been filled in before. What was the order of the fields filled in? How many orders are possible? (Use the notation $a1, a2, \dots, c2, c3$ to refer to individual fields.) **K. 726.** Arrange the numbers $1, 2, 3, 4, \dots, 31, 32$ along the circumference of a circle such that the sum of any pair of adjacent numbers should be a perfect square. Explain your reasoning. **K/C. 727.** On each field of an $n \times n$ table there is a coin, on all of them "heads" showing on top. In each move, we can turn over exactly three coins in any row or column, changing heads to tails and tails to heads. If $n > 2$, is it possible to achieve with a sufficient number of moves that tails should show on top of each coin? Explain your answer. **K/C. 728.** We have 10 cards, with the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 written on them. The cards are laid on the table in a random order in a row, and then we write the number of its position on each card (that is, the cards are numbered from 1 to 10). Thus there will be two numbers on each card. The two numbers on each card are multiplied together, and the products are all added up. *a*) What is the smallest possible value of the final sum? *b*) What is the largest possible value of the final sum?

New exercises for practice – competition C (see page 156): **Exercises up to grade 10: K/C. 727.** See the text at Exercises **K. K/C. 728.** See the text at Exercises **K. Exercises for everyone: C. 1709.** The integers a and b are factors of 720, but ab is not a factor of 720. How many such ordered pairs $(a;b)$ are there? (Proposed by *S. Róka,* Nyíregyháza) **C. 1710.** Four circles are drawn in a unit square as shown in the *figure.* The two larger circles have the same size, and they are tangent to each other as well as to the sides of the square. The two smaller circles are also congruent, and they are also tangent to the sides of the square and to the larger circles. What is the area of the rhombus formed by the centres of the four circles? **C. 1711.** Solve the equation $\sqrt{x-1801}+\sqrt{y-1860}=2-\frac{1}{\sqrt{x-1801}}$, where x and y denote real numbers. **Exercises upwards of grade 11: C. 1712.** The sides of a pentagon are all equal in length, and two of its angles are right angles. What may be the measures of the other three angles? (Proposed by *G. Károlyi,* Budajenő) **C. 1713.** Let x and a denote real numbers such that $x+\frac{1}{x}=a$. Determine the value of $x^{13}+\frac{1}{x^{13}}$ as a function of a.

New exercises – competition B (see page 157): **B. 5230.** Points C and D lie on a semicircular arc of diameter AB. Let A' and B' denote the feet of the perpendiculars dropped to the line CD from the points A and B, respectively. Prove that the line segments $A'C$ and $B'D$ are equal in length. (*3 points*) (Proposed by *L. Surányi,* Budapest) **B. 5231.** Prove that $\sum_{k=1}^{n} k\cdot 2^{k-1}=\sum_{k=1}^{n} 2^{n-k}\cdot(2^k-1)$ for all positive integers n. (*4 points*) **B. 5232.** P is an interior point lying on the median drawn from vertex C of an acute-angled triangle ABC such that $\angle APB=180^\circ-\angle ACB$. Show that the line AB is tangent to the circle APC. (*4 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5233.** The vertices of a regular hexagon are labelled $1, 2, \dots, 6$ in a random order. Then the absolute value of the difference of the labels of the adjacent endpoints is written on each side of the hexagon. Find the expected value of the sum of the six numbers written on the sides. (*4 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5234.** A positive integer n is defined to be a *mythical* number if each of its divisors is 2 smaller than a prime number. For example, 15 is a mythical number. What is the largest possible number of divisors that a mythical number may have? Find all mythical numbers that have the maximum number of divisors. (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5235.** Show that all prime numbers greater than 3 that occur in the Fibonacci sequence are of the form $4k+1$. (*5 points*) **B. 5236.** Let a, b, c denote positive real numbers such that $abc=1$. Show that $a+a^2+a^3+b+b^2+b^3+c+c^2+c^3 \leqslant (a^2+b^2+c^2)^2$. (*6 points*) (Proposed by *M. Lovas,* Budapest and *M. Rozenberg,* Israel) **B. 5237.** In a triangle, r denotes the inradius, R is the circumradius, and s denotes the semiperimeter. Prove that if $r+2R=s$ then the triangle is right angled. (*6 points*) (Proposed by *R. Fridrik,* Szeged)

New problems – competition A (see page 159): **A. 821.** *a*) Is it possible to find a function $f:\mathbb{N}^2\to\mathbb{N}$ such that for every function $g:\mathbb{N}\to\mathbb{N}$ and positive integer m there exists $n\in\mathbb{N}$ such that set $\{k\in\mathbb{N}: f(n,k)=g(k)\}$ has at least m elements? *b*) Is it possible to find a function $f:\mathbb{N}^2\to\mathbb{N}$ such that for every function $g:\mathbb{N}\to\mathbb{N}$ there exists $n\in\mathbb{N}$ such that set $\{k\in\mathbb{N}:\ f(n,k)=g(k)\}$ has an infinite number of elements? **A. 822.** Is it possible to find rational numbers p, q and r such that $p+q+r=0$ and $pqr=1$? (Proposed by *Máté Weisz,* Cambridge) **A. 823.** For positive integers n consider the lattice points $S_n=\{(x,y,z):\ 1\leqslant x\leqslant n,\ 1\leqslant y\leqslant n,\ 1\leqslant z\leqslant n,\ x,y,z\in\mathbb{N}\}$. Is it possible to find a positive integer n for which it is possible to choose more than $n\sqrt{n}$ lattice points from S_n such that for any two chosen lattice points at least two of the coordinates of one is strictly greater than the corresponding coordinates of the other? (Proposed by *Endre Csóka,* Budapest)

Problems in Physics

(see page 186)

M. 412. Place several muffin cupcake paper cases into each other and carry out drop experiments. Measure how the terminal speed of the cases depends on the number of cases. Determine the drag coefficient of the cases.

G. 773. The Earth–Moon system revolves about the centre of mass of the two celestial bodies with a period of 27.32 days, relative to the distant fixed stars. However, in comparison, more than two days more time elapses between two full Moons, namely an average of 29.53 days. Explain the difference between the two period values, and with a simplified calculation show that the difference between them is indeed about two days.

G. 774. The *diagram* on page 187 shows the surface flow velocity profile for the river Danube at bridge Erzsébet on 10 Marc 2018. On the horizontal axis the distance s in metres measured from the left riverbank, and on the vertical axis the speed of the water v in m/s can be seen. The attached *table* shows the recorded data. Estimate the distance at which the river would drift our boat down, if we were to row at a constant velocity of 1 m/s, perpendicularly to the riverbank, to a ship which is at a distance of 221 metres from the left riverbank. **G. 775.** In a thermally insulated flask of negligible heat capacity, there is 1 kg very cold crushed ice, to which 1 kg hot water of temperature 100 °C is poured. What was the temperature of the ice originally if finally there are 2 litres of water at a temperature of 0 °C in the flask? **G. 776.** Watch glasses are to be sterilized in a research laboratory by using UV light. In the sterilization process UV light of total energy of 150 mJ should fall onto a 1 cm^2 surface area of the watch glass. Estimate how long should the UV lamp be operated for this, if

s [m]	v [m/s]	s [m]	v [m/s]
0	0.00	119	1.21
17	0.41	136	1.14
34	1.00	153	1.17
51	1.05	170	1.17
68	1.15	187	1.10
85	1.19	204	1.07
102	1.26	221	1.02