put on the stick, from where it can fall off very easily? **P. 5385.** To what fraction does the heat flux released through a window, which has a single glass layer in it, decreases if the window is replaced by a double pane one? The thickness of each glass layer in both cases is $d_{\text{glass}} = 3$ mm, and in the case of the double pane window there is a $d_{\text{air}} = 7$ mm air gap between the two glass layers. The thermal conductivity of air is $\kappa_{\text{air}} = 0.025$ W/(m K) and the thermal conductivity of glass is $\kappa_{\text{glass}} = 1.2$ W/(m K). **P. 5386.** There is a small ball of charge $Q = 5.55$ μC fixed at the bottom of a 2 m long trough made of some insulating material. The trough makes an angle of elevation of $\alpha = 30^\circ$ with the horizontal. From the top of the trough another small ball of mass $m = 100$ g with charge $q = 10$ μC is released from rest. How far does this ball can move if it rolls without slipping? (The charge of the ball does not change during its motion.) **P. 5387.** Different resistors of resistances R are connected to a battery of electromotive force U_0 and of internal resistance R_{b}. *a*) What is the maximum "useful" power (dissipated in the external resistor) that this battery can deliver? At what external resistance value R can we achieve this maximum power $P_{\max}$? *b*) Show that for any other power P which is smaller than $P_{\max}$, there are two external resistors of resistances $R_1 \neq R_2$ at which the dissipated power is P. What is the arithmetic and the geometric mean of R_1 and R_2? *c*) What is the sum of the terminal voltages across the battery in the above two cases? *d*) What is the sum of the currents through R_1 and R_2? *e*) The efficiency of the delivered energy is defined as the ratio of the useful power to the total power delivered by the battery. What is the sum of the efficiencies in the above two cases? **P. 5388.** Linearly polarized light from a 15 mW laser having a wavelength of $\lambda = 632.8$ nm is emitted from the 2 mm diameter circular aperture of the laser box. *a*) What is the maximum value of the electric field in the laser beam? *b*) What is the total linear momentum of a one metre long piece of the laser beam? **P. 5389.** A (point-like) fly flies at a constant speed of v parallel to the principal axis of a lens, having a focal length of f, at a distance of d from it. What is the least speed of the fly with respect to its image? **P. 5390.** There is a small electric dipole of dipole moment p at the centre of the thin-walled uncharged metal spherical shell of radius R, shown in the *figure*. Determine the surface charge density at points A and B, which are interior points of the shell. Determine the surface density of the charge on the outer surface of the shell as well. (*Hint*: Use the method of image charges applied for a sphere. It might be also useful to know the electric field due to a dipole at a point on the axial and equatorial lines.)

Problems of the 2021 Kürschák competition

1. In the Cartesian coordinate system of the plane, the triangle determined by the points $P_i = (a_i, b_i)$ $(i = 0, 1, 2)$ contains the origin $O = (0, 0)$ in its interior. Show that the areas of the triangles P_0OP_1, P_0OP_2, P_1OP_2 (in this order) form a geometric sequence if and only if the system of equations $a_0x^2 + a_1x + a_2 = b_0x^2 + b_1x + b_2 = 0$ has a real solution x.

2. In Wonderland, n airlines operate flights between n cities. For each airline, there are an odd number of cities, say, $v_1, v_2, \dots, v_i$ such that the airline operates the following flights: v_jv_{j+1} and $v_{j+1}v_j$ for $1 \leqslant j \leqslant i$, where $v_{i+1} = v_i$. Prove that we may choose an odd number of cities, say, $u_1, u_2, \dots, u_k$ in such a way that it is possible to buy tickets for the flights $u_1u_2, u_2u_3, \dots, u_{k-1}u_k, u_ku_1$ from pairwise different airlines.

3. In the cyclic hexagon $A_1B_3A_2B_1A_3B_2$, the diagonals A_1B_1, A_2B_2 and A_3B_3 are concurrent. For $i = 1, 2, 3$, let C_i be the intersection of the diagonals A_iB_i and $A_{i+1}A_{i+2}$, and let D_i be a point on the circumscribed circle, different from B_i, such that the circle $B_iC_iD_i$ is tangent to the line $A_{i+1}A_{i+2}$. (The points are indexed modulo 3, that is, $A_4 = A_1$ and $A_5 = A_2$.) Prove that the segments A_1D_1, A_2D_2 and A_3D_3 are concurrent.