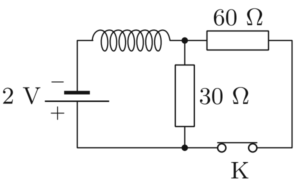
P. 5378. Az *ábrán* látható áramkör K kapcsolója hosszú ideje zárva van. Egyszer csak a kapcsolót kinyitjuk. Mekkora a tekercsben indukálódó feszültség nagysága közvetlenül a kapcsoló kinyitása után?



(*5 pont*) *Példatári feladat nyomán*

P. 5379. Ideális polárszűrők segítségével szeretnénk a lineárisan polarizált fény polarizációs síkját 45°-kal elforgatni úgy, hogy az intenzitásveszteség legfeljebb 10% legyen. Legalább hány polárszűrőre van szükségünk, és hogyan kell azokat optimálisan elhelyezni?

(*5 pont*) *Példatári feladat nyomán*

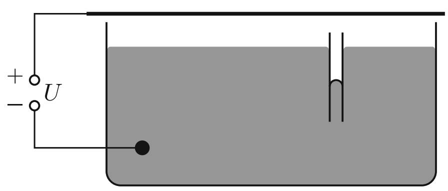
P. 5380. Egy speciális izotóplaborban a doziméterek hitelesítésére extrém aktivitású ^{137}Cs, illetve ^{60}Co forrásokat használnak. A két nagy tisztaságú radioaktív forrás ellenőrzésekor azt tapasztalták, hogy a 68 mg-nyi cézium és egy ismeretlen tömegű kobaltforrás esetében is jó közelítéssel percenként ugyanannyi bomlás történt.

a) Mekkora a kobaltforrás tömege?

b) Mennyi idő múlva és melyik izotópminta aktivitása lesz a másik kétszerese? (A ^{137}Cs felezési ideje: 30,17 év, a ^{60}Co felezési ideje: 5,27 év.)

(*4 pont*) Közli: *Kis Tamás,* Heves

P. 5381. Egy üvegből készült (szigetelő) edény higannyal van töltve. A higanyba egy függőleges, $d = 0{,}5$ mm átmérőjű kapilláris cső merül az *ábrán* látható módon. A higany felszíne fölé $h = 6$ mm magasságban egy nagy kiterjedésű, vízszintes fémlemezt helyeztünk. Mennyivel változik meg a kapilláris csőben a higanyszint, ha a fémlemez és a higany közé $U = 20$ kV egyenfeszültséget kapcsolunk?



(*6 pont*) Közli: *Vigh Máté,* Biatorbágy

Beküldési határidő: 2022. február 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 72. No. 1. January 2022)

Problems in Mathematics

New exercises for practice – competition K (see page 30): **K. 714.** The first term of a sequence is 3, and every further term is obtained by subtracting 2 from the double of the previous term. *a*) List the first 8 terms of the sequence. *b*) Which of the numbers below occur as terms of the sequence, and which of them do not occur? If a number occurs as a term, find the index of the term, otherwise explain why it is not a term of the sequence. 8194, 649 287 365, 29 453 759 372, 8 398 507 839 348. **K. 715.** We have two jugs. Each of them can hold 2 litres of liquid. Initially, one jug is filled with 2 litres of 100% orange juice, and the other contains 1 litre of water. 1. Half the orange juice is poured into the water jug, and stirred with a spoon. Then 1 litre of the mixture is poured back in the first jug. 2. The procedure is repeated once more: 1 litre is transferred from the first jug to the second, stirred, and 1 litre is transferred back to the first jug. Find the resulting percentage of orange juice in the content of each jug. **K. 716.** In a shop, three notebooks and two pens cost 1110 forints (HUF, Hungarian currency). Five notebooks and four pens cost 2010 forints. What is the price of one notebook, and what is the price of one pen? **K/C. 717.** In a regular dodecagon $ABCDEFGHIJKL$ the squares $ABPQ$ and $GHRS$ are drawn on sides AB and GH, on the inside, as shown in the *figure.* Show that PQ and RS are two opposite sides of a regular hexagon. **K/C. 718.** How many numbers are there from 1 to 50 that can be represented as a sum of at least two consecutive non-negative integers?

New exercises for practice – competition C (see page 31): **Exercises up to grade 10: K/C. 717.** See the text at Exercises **K. K/C. 718.** See the text at Exercises **K. Exercises for everyone: C. 1699.** In the expansion of the product $(x+1)\cdot(x^2+1)\cdot(x^3+1)\cdot\ldots\cdot(x^{12}+1)$, what is the coefficient of the term in x^{14}? **C. 1700.** In a circle of centre O, A is an interior point different from O. For a point B on the circumference of the circle, $\angle OAB=\alpha$. Let C be a point on the circumference such that $2\alpha+\beta=180^\circ$, where $\angle BAC=\beta$, and the angles $\angle BAO$ and $\angle BAC$ have no common points apart from the ray AB. Prove that the points O, A, B, C are concyclic. **C. 1701.** What is the sum of all integers x for which $\sqrt{2x^2-6x-20}<-x+5$? **Exercises upwards of grade 11: C. 1702.** Vertex A of a quadrilateral $ABCD$ lies on the plane S, its diagonal BD is parallel to the plane S, and its vertex C is at a distance of 8 units from the plane S. Given that the orthogonal projection of the quadrilateral onto S is a square with a diagonal 6 units long, prove that quadrilateral $ABCD$ is a rhombus, and calculate the length of its sides. (Proposed by *N. Zagyva,* Baja) **C. 1703.** The natural numbers a and b each have only digits of 1 in decimal notation. Prove that if a and b are not relatively prime then the sums of their digits, $S(a)$ and $S(b)$ are not relatively prime either.

New exercises – competition B (see page 32): **B. 5214.** The sequence of digits 110 represents an even integer, whatever positive integer greater than 1 is the base of notation. Is there a sequence of digits 1 and 0 such that it represents a multiple of 3, whatever positive integer greater than 1 is the base of notation? (*3 points*) **B. 5215.** Find all positive real numbers x for which $x+\frac{1}{x}$ is an integer, and $x^3+\frac{1}{x^3}$ is a prime number. (*4 points*) (Based on the idea of *B.* and *V. Szaszkó-Bogár*) **B. 5216.** The tangents drawn to the circumscribed circle of a right triangle ABC at the right-angled vertex C and at another vertex A intersect at D. Prove that the line BD bisects the altitude drawn from vertex C. (*3 points*) **B. 5217.** A new triangle is constructed out of the line segments obtained by multiplying the medians of a triangle by $\frac{2}{\sqrt{3}}$. The procedure is repeated with the triangle obtained. Show that the triangle obtained in the second step is congruent to the original triangle. (*4 points*) (Proposed by *P. Bártfai,* Budapest) **B. 5218.** What is the largest number of elements that can be selected out of the first 2022 positive integers such that the difference of any two selected numbers is not a prime? (*5 points*) **B. 5219.** Prove that $\frac{|a+b+c|}{1+|a+b+c|}\leqslant\frac{|a|}{1+|a|}+\frac{|b|}{1+|b|}+\frac{|c|}{1+|c|}$ for all real numbers a, b, c. On what condition will equality occur? (*5 points*) (Proposed by *J. Schultz,* Szeged) **B. 5220.** Let n be a positive integer. Prove that it is possible to select n perfect squares from the numbers 1 to 2^{n+2} such that the sums obtained by adding an arbitrary subset of the selected numbers (including sums of single terms and the sum of all the numbers) are all distinct. (*6 points*) (Proposed by *R. Freud,* Budapest) **B. 5221.** In an acute-angled triangle ABC, the points of tangency of the inscribed circle on sides BC, CA, AB are D, E, and F, respectively. The circumscribed circle of the triangle intersects circle AEF at a point P different from A, intersects circle BFD at a point Q different from B, and intersects circle CDE at a point R different from C. Show that the lines DP, EQ and FR are concurrent. (*6 points*) (Proposed by *M. Lovas,* Budapest)

New problems – competition A (see page 34): **A. 815.** Let q be a monic polynomial with integer coefficients. Prove that there exists a constant C depending only on polynomial q such that for an arbitrary prime number p and an arbitrary positive integer $N \leqslant p$ the congruence $n! \equiv q(n) \pmod{p}$ has at most $CN^{2/3}$ solutions among any N consecutive integers. (Submitted by *Navid Safaei,* Iran) **A. 816.** Peter has 2022 pieces of magnetic railroad cars, which are of two types: some has the front with north and the rear with south magnetic polarity, and some has the rear with north and the rear with south magnetic polarity (on these railroad cars the front and the rear can be distinguished). Peter wants to decide whether there are the same number of both type of cars. He can try to fit together two cars in one try. What is the least number of tries needed? (Submitted by *Dömötör Pálvölgyi,* Budapest) **A. 817.** Let ABC be a triangle. Let T be the point of tangency of the circumcircle of triangle ABC and the A-mixtilinear incircle (the circle which is tangent to sides AB, AC, and internally tangent to the circumcircle of triangle ABC). The incircle of triangle ABC has center I and touches sides BC, CA and AB at points D, E and F, respectively. Let N be the midpoint of line segment DF. Prove that the circumcircle of triangle BTN, line TI and the perpendicular from D to EF are concurrent. (Submitted by *Diaconescu Tashi,* Romania)

Problems in Physics

(see page 58)

M. 410. If a playing card is placed between a small, strong magnet and a horizontal paper clip, then we can lift the paper clip by the card. Measure how many cards you need to stack so that you can no longer lift the paper clip. What is the thickness of these stacked sheets? Connect two magnets of the same size and investigate how many cards are needed so that we can no longer lift the paper clip.

G. 765. Each of the 11 shots in the series of photos was taken from the same location, with the camera always facing the Sun. The chronological order of the photos is from left to right. What was the time when the image of the Sun was the closest to the horizon? To which point of the compass did the camera face when the Sun was at its lowest position in the sky? Where and in which season was the series of photos taken? **G. 766.** The most famous formula in physics is the relation of $E = mc^2$ expressing the equivalence of mass and energy where E is the energy, m is the mass, and c is the speed of light in vacuum. Using this, estimate how much heavier is our mobile phone, when its battery is fully charged, than when its battery is fully discharged. **G. 767.** The Morning Star (actually the planet Venus) is visible for a while only in the evenings, and then it appears for a while only at dawns. What is the period of this change? **G. 768.** The *figure* shows the current-voltage characteristic of one of the tungsten filament bulbs of a string of a Christmas tree lights, which contains a hundred bulbs connected in series. *a*) Using the graph determine the total dissipated electrical energy by all the bulbs of the string if it is connected to a voltage supply of 230V. *b*) What is the total dissipated electrical energy by the string if it contains only ten bulbs and it is connected to 230 V? *Note:* In the second case the bulbs will quite soon burn out.