P. 5371. A tau-részecske (τ) elektromos töltése ugyanakkora, mint az elektroné. Tömege 3470-szer akkora, mint az elektroné és 1,89-szer akkora, mint a protoné. Nagyon rövid az élettartama ($3\cdot 10^{-13}$ s), mégis előfordulhat, hogy a protonnal kötött rendszert alkot. Ebben az esetben a két részecske a közös tömegközéppont körül körpályán kering, és a rendszer teljes perdülete $n\hbar$ ($n=1,2,\ldots$).

a) Adjuk meg a τ-proton atom és a H-atom színképeiben a megfelelő hullámhosszak arányát!

b) Mekkora a τ-proton atom kötési energiája?

(*5 pont*) Közli: *Simon Péter,* Pécs

P. 5372. Egy rúdinga (egyik végénél felfüggesztett homogén rúd) szabad lengéseinek körfrekvenciája ω. Állandósult állapotban mekkora amplitúdójú rezgéseket végez a rúd alsó végpontja, ha az inga felfüggesztési pontját vízszintes irányban $x(t)=A\cos(2\omega t)$ időfüggésű kitéréssel mozgatjuk? (A közegellenállás kicsi, de nem teljesen elhanyagolható, továbbá $A\omega^2\ll g$.)

(*6 pont*) Közli: *Vigh Máté*, Biatorbágy

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Beküldési határidő: 2022. január 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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Problems in Mathematics

New exercises for practice – competition K (see page 542): **K. 709.** A family are planning to grow a special type of onion in their garden. They want to eat 400 onion bulbs each year. The onion is grown from a seed. Each onion plant can produce 51 seeds every year if we let it. However, once an onion plant has gone into seeds, its bulb will dissolve during growing the seeds. What is the minimum number of seeds the family need to buy in order to launch the onion project so that they can have the desired quantity of onion bulbs to eat, and they never need to buy seeds any more? **K. 710.** There are some dodecahedra and some icosahedra on the table. The solids have 792 vertices and 936 faces altogether. How many dodecahedra and how many icosahedra are there on the table? **K. 711.** Ann's favourite number is 2468. Ben's favourite number also has four digits, and exactly two of its digits coincide with two digits of Ann's favourite number. Furthermore, each of the matching digits is in the same decimal place in both numbers. Based on this information, how many four-digit positive integers are there which may be Ben's favourite number? **K/C. 712.** We have 2022 square wooden plates arranged in a row, and 2021 discs numbered 1 to 2021. One disc is placed on each wooden plate except for the last one, in a random order. Then in each move, one disc is transferred to the (momentarily) vacant wooden square. Our goal is to have the numbers in increasing order, with the last square left vacant. What is the maximum number of moves that may be needed to achieve that goal? Also give an example for a possible initial arrangement that does require that maximum number of moves. **K/C. 713.** The sides of a square are 6 cm long. A circle is drawn over each side as diameter, as shown in the *figure.* The large circle is centred at the centre of the square and its radius equals the side of the square. Calculate the areas of the regions I., II. and III.

New exercises for practice – competition C (see page 543): **Exercises up to grade 10: K/C. 712.** See the text at Exercises **K. K/C. 713.** See the text at Exercises **K. Exercises for everyone: C. 1694.** Solve the following equation over the set of real numbers:

$$x-2021-\frac{x-2020}{2}+\frac{x-2019}{3}-\frac{x-2018}{4}+\frac{x-2017}{5}-\dots+$$
$$+\frac{x-3}{2019}-\frac{x-2}{2020}+\frac{x-1}{2021}-\frac{x}{2022}=0.$$

(Proposed by *L. Sáfár,* Ráckeve) **C. 1695.** A perpendicular line segment equal in length to the radius (R) is attached to the circumference of a circle (as shown in the *figure*). Is it possible to fit two such circles with handles ("frying pans") in a rectangle if one side of the rectangle equals the diameter of the circle ($2R$), and the other side is twice as long ($4R$)? The figures and the rectangle are allowed to touch one another but they are not allowed to intersect or overlap. (Proposed by *M. E. Gáspár,* Budapest) **C. 1696.** There are 10 and 15 points marked on two parallel lines a and b, respectively. Consider all line segments formed by the points marked such that one endpoint lies on a, and the other lies on b. How many intersections may these line segments have altogether at most? **Exercises upwards of grade 11: C. 1697.** Interior points D, E, F are marked on sides BC, CA, AB of a regular triangle ABC, respectively, such that DEF is also a regular triangle. Then regular triangles BDA' and AEB' are drawn over the sides BC and AC, respectively, on the outside. Prove that there exists a point on side AB where each of the line segments $A'E$ and $B'D$ subtends a right angle. **C. 1698.** Zoe does not like books but she decides to read a total of 2021 pages during the year 2021. She is going to read on consecutive days, one page more on every day than on the previous day. How many pages should she read on the first day if she would like to spread the project through the largest possible number of days, but she does not have time to read more than 100 pages on any day? (Proposed by *L. Sáfár,* Ráckeve)

New exercises – competition B (see page 544): **B. 5206.** An n-digit number $\overline{a_1a_2a_3\dots a_n}$ is called *hill type* if there exists an integer $1\leqslant k\leqslant n$ for which the sequence $a_1,a_2,\dots,a_k$ is strictly increasing and the sequence $a_k,a_{k+1},\dots,a_n$ is strictly decreasing. (For example, the numbers 1, 121, 1231 are of hill type, whereas 1442 or 12313 are not.) How many hill type numbers are there? (*3 points*) **B. 5207.** Let $n\geqslant 2$ be a natural number. Prove that there exist positive integers $2\leqslant x_1<x_2<x_3<\dots<x_n$, such that $x_1!\cdot x_2!\cdot x_3!\cdot\dots\cdot x_n!$ is a perfect square. (*4 points*) **B. 5208.** The lines of two perpendicular chords AB and CD of a circle intersect at an exterior point P. The length of the tangent drawn from P to the circle is e. Show that the geometric mean of the lengths AD and BC is at least $\sqrt{2}\,e$. (*4 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5209.** What is the maximum possible number of two-element subsets in a 2022-element set of integers for which the sum of the two elements also belongs to the set? (*5 points*) **B. 5210.** The parabolas $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ have the same focus, and any two of them intersect at exactly two points. Let e_{ij} denote the line passing through the two intersections of the parabolas $\mathcal{P}_i$ and $\mathcal{P}_j$. Show that the lines e_{12}, e_{13} and e_{23} are concurrent. (*5 points*) **B. 5211.** Solve the

following equation over the set of positive integers: $5^x - 2^y = 1$. (*5 points*) **B. 5212.** Prove that there exists a positive integer which can be represented in at least 2021 different ways by taking an appropriate positive integer (in decimal notation), and adding the sum of its digits to it. (*6 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5213.** Prove that if a, b, c are positive real numbers, then

$$c\sqrt{a^2+b^2-ab}+a\sqrt{b^2+c^2-bc}\geqslant b\sqrt{c^2+a^2+ca}\,.$$

When will equality occur? (*5 points*) (Proposed by *J. Schultz,* Szeged)

New problems – competition A (see page 546): **A. 812.** Two players play the following game: there are two heaps of tokens, and they take turns to pick some tokens from them. The winner of the game is the player who takes away the last token. If the number of tokens in the two heaps are A and B at a given moment, the player whose turn it is can take away a number of tokens that is a multiple of A or a multiple of B from one of the heaps. Find those pair of integers (k,n), for which the second player has a winning strategy, if the initial number of tokens is k in the first heap and n in the second heap. (Proposed by *Dömötör Pálvölgyi,* Budapest) **A. 813.** Let p be a prime number and k a positive integer. Let $t=\sum_{j=0}^{\infty}\left\lfloor\frac{k}{p^j}\right\rfloor$. *a*) Let $f(x)$ be a polynomial of degree k with integer coefficients such that its leading coefficient is 1 and its constant is divisible by p. Prove that there exists $n\in\mathbb{N}$ for which $p\mid f(n)$, but $p^{t+1}\nmid f(n)$. *b*) Prove that the statement above is sharp, i.e. there exists polynomial $g(x)$ of degree k, integer coefficients, leading coefficient 1 and constant divisible by p such that if $p\mid g(n)$ is true for a certain $n\in\mathbb{N}$, then $p^t\mid g(n)$ also holds. (Proposed by *Kristóf Szabó,* Budapest) **A. 814.** There are given 666 points in the plane such that they cannot be covered with 10 lines. Prove that it is possible to choose 66 of them such that they still cannot be covered with 10 lines. (Proposed by *Mihály Hujter,* Budapest)

Problems in Physics

(see page 570)

M. 409. Let us make a sponge cake. Measure the density of the dough before baking and after baking. Investigate how the density of the finished sponge cake varies depending on whether it was baked on the edge of the cake pan or in the middle. (Enter the sponge cake recipe as well.)

G. 761. How was the word "HÁTULJA" (meaning BACK) written on the back of KöMaL: as usual or in mirror writing? **G. 762.** The *photo on the left* shows a garden solar shower. The incident sunlight heats up the water in the vertical black container. From the lower tap (the so-called leg wash) only cold water flows, while from the upper shower rose we can enjoy the water jet, whose temperature can be set with a single-lever shower mixer positioned in the middle. For the operation of the shower cold water flows in at the bottom. The structure of a shower without a foot wash is shown in the *figure on the right.* Complete the diagram with a foot wash, then explain how the garden shower works. **G. 763.** We have two solid cubes, one made of aluminium and the other made of copper. Placed on a particularly accurate scale they are both measured to be 1 ton to the nearest gram in vacuum. What will the difference between the results of the measurements be if the masses of the cubes are measured in air at standard temperature and pressure (STP)? **G. 764.** An object was released from rest and falls freely. It covered the same distance in the last second of its motion as it did in the first three seconds. From what height did it fall? (Neglect air resistance.)