four against four, on each of the first seven days of the autumn break. Is it possible to organize the teams so that any set of three boys would play in the same team at least once? (*5 points*) (Based on the idea of *M. E. Gáspár,* Budapest) **B. 5193.** In an acute-angled triangle ABC, $\angle BCA = 45^\circ$, the feet of the altitudes on sides BC, CA, AB are D, E, F, respectively, and the orthocentre is M. Point F divides line segment AB in a ratio $AF : FB = 2 : 3$. G is the point on side AC for which $CG = BM$. Show that the centroid of triangle ABG is M. (*4 points*) **B. 5194.** In a triangle ABC, $\angle ABC = 2\angle CAB$. Side AB touches the inscribed circle at point E, and intersects the angle bisector drawn from C at point F. Prove that $AF = 2BE$. (*4 points*) **B. 5195.** Prove that the inequality $x^p \cdot y^{1-p} < x + y$ holds for every pair of positive real numbers (x, y), and all real numbers $0 < p < 1$. (*3 points*) **B. 5196.** Let $p(x) = 2x + 1$. A is a subset of set $S = \{1, 2, \ldots, 2021\}$ such that it contains at most one of the numbers n, $p(n)$, $p\big(p(n)\big)$ for every n, but this condition will not hold anymore if any extra element of S is added to A. What may be the number of elements in the set A? (*6 points*) **B. 5197.** Let $\mathbb{N}$ denote the set of non-negative integers, and let k be a given positive integer. Is there a monotonically increasing function $f : \mathbb{N} \to \mathbb{N}$ such that $f\big(f(x)\big) = f(x) + x + k$ for all $x \in \mathbb{N}$? (*6 points*)

New problems – competition A (see page 415): **A. 806.** Four distinct lines are given in the plane, which are not concurrent and no three of which are parallel. Prove that it is possible to find four points in the plane, A, B, C and D with the following properties: (*i*) A, B, C and D are collinear in this order; (*ii*) $AB = BC = CD$; (*iii*) with an appropriate order of the four given lines A is on the first, B is on the second, C is on the third and D is on the fourth line. (Proposed by *Kada Williams,* Cambridge) **A. 807.** Let $n \geqslant 2$ be a given integer. Let G be a finite simple graph with the property that each of its edges is contained in at most n circuits. Prove that the chromatic number of the graph is at most $n + 1$. (Proposed by *Ádám Schweitzer,* Budapest) **A. 808.** Find all triples of positive integers a, b and c such that a, b and c are pairwise relatively prime and $a^2 + 3b^2c^2 = 7^c$. (Proposed by *Nikolai Beluhov,* Bulgaria)

Problems in Physics

(see page 443)

M. 407. An EPS panel is a set of compressed small styrofoam balls. If the panel is broken or sawn, such small balls can easily fall out of it. Measure how many times the density of some of these pellets is greater than the density of the EPS panel.

G. 753. Two 5-metre long vehicles are travelling one after the other on a highway at a speed of 100 km/h. The distance between the cars is 30 m. Once, the car at the back starts overtaking. It accelerates uniformly until the two cars are next to each other. At this moment the speed of the accelerating car is 130 km/h, which remains constant for the rest of the motion. This car finishes the overtaking manoeuvre by positioning itself 30 m ahead of the other car moving at a constant speed. How long did the overtaking last? **G. 754.** Newspaper news on March 20, 2021: "The vast majority of space debris revolve around the Earth at low orbits, i.e. from an altitude of 800 km up to 2000 km, at a speed of 28 000 km/h." *a*) At what altitude can a piece of space debris orbit at a speed of 28 000 km/h? *b*) At what speed can a piece of space debris orbit at an altitude between 800 to 2000 kilometres? **G. 755.** An 80 kg action hero uses a parachute that sinks at a speed of 8 m/s when open. In one scene, he catches the heroine, who weighs 60 kg, in the air and then he opens the parachute. At what speed does the clinging pair reach the ground? From what height should they jump without parachute in order to reach the ground at the same speed? **G. 756.** The gauge pressure in the tyre of a car measured by a meter at a gas station is 1.2 bars. Assuming that neither the volume of the tyre nor the temperature of the air in the tyre change, by what percentage does the number of molecules in the tyre increase if the pressure is increased to the required 2.4 bars?

P. 5346. We move uniformly with a wheelbarrow, which has a big wheel and weighs $G = 100$ N, along the level ground. In this case we have to exert vertically upward forces of magnitude 25 N at the end of each handle of the wheelbarrow. By means of a ruler and a protractor construct and determine the magnitude and the direction of the force $\boldsymbol{F}$ exerted on each handle if we move along a slope of angle of elevation $\alpha = 18^\circ$ downward then upward. Verify the result by calculation. For the sake of simplicity assume that the distance between the ground and the ends of the rods is always the radius of the wheel, and that the centre of mass of the wheelbarrow is also at this distance from the ground. **P. 5347.** An initially stationary object of mass $m = 2$ kg can move frictionlessly along a horizontal surface. At a certain moment a force of constant direction and parallel to the surface is started to be exerted on the object. The magnitude of the force is increasing uniformly from 0 to 20 N in 4 s. *a*) What will the speed of the object be after $t_1 = 3$ s? *b*) How much distance does the object cover in 3 s, if the distance covered in $t_2 = 2$ s is $s_2 = \frac{10}{3}$ m? **P. 5348.** In a demonstration flight, a new passenger aircraft travelled at a speed of 85.2 m/s at a height of 150 metres, where the temperature of the air was $15\,^\circ$. This speed is one quarter of the speed of sound there, which is usually formulated as $v = 0.25$ M, i.e. 0.25 Mach. At ground level, the air had a temperature of $16\,^\circ$C. The cruising speed of this aircraft is 900 km/h, which is 0.82 M (0.82 Mach) at the cruising altitude, and at the temperature there. Considering air as an ideal gas and assuming that the temperature of the air varies linearly with the distance measured from the ground, determine *a*) the temperature of the air at the cruising height; *b*) the cruising height. **P. 5349.** A battery of internal resistance 1.5 Ω is connected to two resistors connected in parallel, one of them having a resistance of $R_1 = 40$ Ω and the R_2 resistance of the other resistor is not known. Determine the unknown resistance of the second resistor if it dissipates 60% of the total energy delivered by the battery. **P. 5350.** A thin parallel light beam is aimed at the centre of a transparent sphere, and the rays meet exactly at the opposite point of the surface of the sphere. What is the refractive index of the material of the sphere? **P. 5351.** Why is it not allowed to look into a laser light? The lens in the human eye can focus the light rays into a very small spot of diameter of several μm. The most sensitive cells are in the retina, and here the size of the cells so called rods and cones is in the range of μm. The power of lasers used in everyday life is between 0.1 mW and 100 mW. Calculate how long it takes for the light of the smallest laser, i.e. 0.1 mW, with 80% light absorption, to heat a cell to $50\,^\circ$C at which the cell gets damaged, and how long it takes to heat the cell to a temperature of $100\,^\circ$C at which the cell is totally destroyed. For the sake of simplicity consider the cell as a cylinder which has a base diameter of 5 μm and a height of 7 μm; the density and the specific heat capacity of the cell can be assumed to be the same as those of water. The temperature of the eye can be considered as $36\,^\circ$C, and other effects (as displacements, heat conduction etc.) can be neglected. Compare the gained time with the approximately 0.2 second reaction time of the human eye. **P. 5352.** A closed circular loop of wire having a resistance of R and enclosing a cross section of A is rotated at a constant angular speed of ω in magnetic field of induction B, about that symmetry axis of the loop which lies in the plane of the loop. At what average power can this be done? **P. 5353.** What is the reason that the activity of mined uranium ore is significantly higher than that of the uranium salt which is made from the ore? **P. 5354.** A toy train, equipped with a motor, travels along a circular track of radius R at a constant speed of v. At a distance of $d < R$ there is a point-like sound source, emitting sound of frequency f_0. A microphone is attached to the train. Determine the range of the detected frequency of the sound. (The speed of sound is c.)