a) Mekkora az α szög, ha $\frac{R}{r}=\frac{5}{2}$?

b) Vizsgáljuk meg, hogy különböző $\frac{R}{r}$ arányoknál mekkora α szög (vagy szögek) esetében valósulhat meg a leírt mozgás!

(*6 pont*) *Romániai versenyfeladat nyomán*

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Beküldési határidő: 2021. október 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 71. No. 6. September 2021)

Problems in Mathematics

New exercises for practice – competition K (see page 351): **K. 694.** How many seven-digit positive integers are there in which each digit is either 1 or 2 greater than the preceding digit? (Example: as in 1 234 678.) **K. 695.** A point P is selected on side BC of a square sheet of paper $ABCD$. The sheet is folded along the line AP so that point B should lie equidistant from vertices C and D. The new position of point B is denoted by B'. Determine the measure of angle $CB'D$. **K. 696.** In the left front pocket of my jeans I have twice as much money as in the right front pocket, and one third as much as in the right back pocket. I moved 30 forints (HUF, Hungarian currency) from the right front pocket to the left front pocket, and also moved 180 forints from the right back pocket to the left front pocket. Now I have 3 times as much money in the left front pocket as the amount remaining in the right front pocket. How much money did I have initially in each pocket? **K. 697.** Some of the faces of a cube are coloured red, and then the cube is cut into small cubes of equal size. 45 of the small cubes have no painted faces. How many faces of the original cube were coloured? **K. 698.** Dorothy thought of an integer that is at least 3 and at most 25. Ann named a one-digit even number x, and asked Dorothy whether her number is a perfect square, whether it is prime, and whether it is a multiple of x. Dorothy said if she gave the answer to each of these questions, Ann would be able to figure out what number she had in mind. What is Dorothy's number?

New exercises for practice – competition C (see page 351): **Exercises up to grade 10: K/C. 697.** See the text at Exercises **K. K/C. 698.** See the text at Exercises **K. Exercises for everyone: C. 1679.** Prove that the value of the expression $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots+\frac{1}{2021}-\frac{1}{2022}$ is between 0 and 1. **C. 1680.** One side of a quadrilateral is 5 cm long, and the measures of the angles lying on it are 90° and 60°. Given that the quadrilateral has both an inscribed circle and a circumscribed circle, find a method to construct the quadrilateral. Write down the steps of the construction. (Elementary steps of construction, like bisecting an angle or reflecting about a line do not need to be described in detail.) (Proposed by *N. Zagyva,* Baja) **C. 1681.** Let a, b, c denote nonzero real numbers that add up to 0. Prove that $\frac{a^3-a^2+b^3-b^2+c^3+c^2}{ab}=3c+2$. **Exercises upwards of grade 11: C. 1682.** The vertices of a unit cube are A, B, C, D, E, F, G, H as shown in the *figure.* The tetrahedra $ABDE$ and $GCFH$ are cut off the cube. Find the volume and surface area of the remaining solid. (Proposed by *N. Zagyva,* Baja) **C. 1683.**

Ann and Bo are playing the following game on squared sheets of paper. Each player marks a 10×10 square on her own sheet of squared paper. In this large square, they colour seven 1×1 lattice squares blue, and another 14 lattice squares red. The players cannot see each other's coloured squares. The game starts by Ann naming a pair of numbers (i,j) where $1\leqslant i,j\leqslant 10$ are positive integers. (For example, $(5,2)$ means the lattice square at the intersection of row 5 and column 2 of the 10×10 square.) If the pair (i,j) determines a coloured field on Bo's sheet of paper then Bo will answer "hit", otherwise she will say "no hit". Then the game continues by switching roles: Bo names a pair of numbers and Ann answers. What is the probability that in the third round of the game Ann will hit a blue square and Bo will hit a red square provided that in the first two rounds neither Ann nor Bo had any hits?

New exercises – competition B (see page 353): **B. 5182.** The number $612^2=374\,544$ ends in two digits of 4 in base 10 notation. What is the maximum number of digits of 4 at the end of a perfect square? (*3 points*) (Based on the idea of *I. Blahota*) **B. 5183.** Side AB of a triangle ABC has unit length, $\angle BAC=60^\circ$, $\angle ACB=100^\circ$ and the midpoint of side BC is F. D is a point on side AB such that $DB=FB$. Find the exact value of $T_{ABC\triangle}+2T_{FBD\triangle}$, where $T_{ABC\triangle}$ denotes the area of triangle ABC. (T means the area of the triangle of the triangle named in the index.) (*4 points*) (Proposed by *S. Kiss,* Nyíregyháza) **B. 5184.** Cornelia marked four non-concyclic points in the plane. Then she drew all the circles that pass equidistant from the four points. What is the maximum possible number of circles she may have drawn? (The distance between point P and a circle k centred at O is defined as follows: let M denote the point where the ray starting at O and passing through P intersects the circle k. Then the distance is the length of line segment PM.) (*5 points*) **B. 5185.** Find the real solutions of the equation $\sqrt[3]{4-x^2}+\sqrt{x^2-3}=1$. (*4 points*) (Proposed by *M. Szalai,* Szeged) **B. 5186.** Al and Bill are playing the following game. They agree on a fixed number $n\geqslant 3$, and then Al thinks of a number from the set $\{1,2,\dots,n\}$. Now Bill can guess the number. He will only get yes or no answers. If the answer is yes, the game terminates. If the answer is no, Al will change the number: either increases or reduces it by 1, but the number must remain positive (it is allowed to go beyond n though). Then Bill can guess again, trying to hit the new number. The procedure is repeated until finally Bill gets the number. Prove that Bill has a strategy to end the game with at most $(3n-5)$ guesses. (*6 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5187.** A subset of the set $S=\{1,2,\dots,n\}$ is called *primitive,* if it does not contain two elements such that one is a divisor of the other. Show that if it is not possible to add a further element of S to a particular primitive set $A\subseteq S$ and keep it primitive, then either $A=\{1\}$ or the size of A is greater than or equal to the number of primes up to n. (*6 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5188.** Prove that the height of a circumscribed trapezium cannot be greater than the geometric mean of the bases. (*5 points*) (Proposed by *L. Németh,* Fonyód) **B. 5189.** The base edge of a right pyramid with a regular triangular base is a. Let r be the radius of the inscribed sphere, and let R be the radius of the escribed sphere touching the base. Prove that $a^2=12rR$. (*6 points*) (Proposed by *L. László,* Budapest)

New problems – competition A (see page 355): **A. 803.** Let $\pi(n)$ denote the number of primes less than or equal to n. A subset of $S=\{1,2,\dots,n\}$ is called *primitive* if there are no two elements in it with one of them dividing the other. Prove that for $n\geqslant 5$ and $1\leqslant k<\frac{\pi(n)}{2}$ the number of primitive subsets of S with $k+1$ elements is greater or equal to the number of primitive subsets of S with k elements. (Proposed by *Cs. Sándor,* Budapest) **A. 804.** There is a city with n citizens. The city wants to buy *sceptervirus* tests with which it is possible to analyze the samples of several people at the same time.

The result of a test can be the following: • Virus positive: there is at least one currently infected person among the people whose samples were analyzed, and none of them were healed from an earlier infection. • Antibody positive: there is at least one person who was healed from an earlier infection among the people whose samples were analyzed, and non of them are infected now. • Neutral: either all of the people whose samples were analyzed are not infected, or there is at least one currently infected person and one person who was healed from an earlier infection. (Viruses and antibodies in samples completely neutralize each other.) What is the smallest number of tests to buy if we would like to know if the sceptervirus is present now or it has been present earlier? (The samples are taken from the people at the same time. The people are either infected now, have been infected earlier, or haven't contacted the virus yet.) (Submitted by *Csongor Beke,* Cambridge) **A. 805.** In acute triangle ABC the feet of the altitudes are A_1, B_1 and C_1 (with the usual notations on sides BC, CA and AB, respectively). The circumcircles of triangles AB_1C_1 and BC_1A_1 intersect the circumcircle of triangle ABC at points $P \neq A$ and $Q \neq B$, respectively. Prove that lines AQ, BP and the Euler line of triangle ABC are either concurrent or parallel to each other. (Submitted by *Géza Kós,* Budapest)

Problems in Physics

(see page 377)

M. 406. At home make an inclined plane of small and variable angle of inclination. Fix the slope at a certain angle of inclination of α. Place a cylinder-shaped pencil on the slope such that the symmetry axis of the pencil makes an angle of β with a horizontal line lying in the plane of the slope. Investigate, at a certain fixed angle of α, at which angle of β will the pencil begin to move along the plane such that it *a*) slides down, but does not roll at all; *b*) rolls down without slipping? Investigate the rolling and slipping regions, and plot them in a coordinate system of (α, β).

G. 749. A flight operated by the United Airlines, UA425 passed right in front of the Sun when *Andrew McCarthy,* a well-known American astrophotographer, took the attached picture. Estimate the distance between the plane and the photographer's camera! **G. 750.** Jack, whose mass is 35 kg, stands on a bathroom scale. He holds a spring balance, which weighs 0.5 N, in his hand, and he hangs a traditional balance to the spring balance. The weight of the empty traditional balance is 15 N. In one plate of the traditional balance there is a stone, which is balanced by weights whose total mass is 2 kg and 20 dag. What is the reading on the bathroom scale? **G. 751.** The image formed by a plane mirror has the same size as the object. However, if we go closer to the mirror we observe ourselves bigger, because the angle of view gets greater. We can see our back by means of two plane mirrors, which are placed approximately opposite and parallel to each other. Where should we stand in between the two mirrors in order to get the greatest angle of view of our back? **G. 752.** The one-ton Mars rover called Perseverance landed successfully on Mars in the middle of February this year. The rover also carried a helicopter drone. What is the weight of the rover on Mars? How could they put the helicopter drone to test on the Earth? Make a suggestion.

P. 5337. Two freight trains are travelling along two parallel railways at a uniform speed. They pass each other in 20 seconds if they move towards each other, while it takes 60 s to pass if they move into the same direction. It takes 40 s for one of the trains and 100 s for the other one to cross a 600-m long bridge. Determine the speeds and the lengths of the trains. **P. 5338.** A pair of dominoes are placed to a third domino as it is shown in the left *figure. a*) Determine the possible values of x such that the dominoes are in stable equilibrium. *b*) Then several more domino pairs are placed to the dominoes as