of leg BC at point P, and the hypotenuse AB at point M. The tangent drawn to circle k at point F intersects the line of leg AC at point Q, and the hypotenuse AB at point N. Prove that $4 \cdot MN^2 = PE^2 + QF^2 + 2 \cdot EF^2$. (*5 points*) **B. 5178.** Let x be a positive real number. Show that $\sqrt{6x+9}+\sqrt{16x+64} \leqslant \left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{8}{\sqrt{x}}\right)$. (*4 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5179.** Is there a set H of integers with the following property: every nonzero integer can be represented in infinitely many ways as a sum of some distinct elements of H, but 0 cannot be represented at all? (*6 points*) **B. 5180.** The radius of the circumscribed circle of a regular heptagon $ABCDEFG$ is r. Prove that the circle of radius $2r$ centred at A passes through the orthocentre of triangle BCE. (*5 points*) **B. 5181.** Given eight points on the plane, no three of which are collinear and no five of which are concyclic, what is the maximum possible number of circles that pass through four of the points each? (*6 points*) (Proposed by *A. Imolay,* Budapest)

New problems – competition A (see page 288): **A. 800.** In a finite, simple, connected graph G we play the following game: initially we color all the vertices with a different color. In each step we choose a vertex randomly (with uniform distribution), and then choose one of its neighbors randomly (also with uniform distribution), and color it to the the same color as the originally chosen vertex (if the two chosen vertices already have the same color, we do nothing). The game ends when all the vertices have the same color. Knowing graph G find the probability for each vertex that the game ends with all vertices having the same color as the chosen vertex. (Submitted by *Dávid Matolcsi,* Budapest) **A. 801.** For which values of positive integer m is it possible to find polynomials $p, q \in \mathbb{C}[x]$ with degrees at least two such that $x(x+1)\cdots(x+m-1) = p\big(q(x)\big)$? (Submitted by *Navid Safaei,* Tehran) **A. 802.** Let P be a given regular 100-gon. Prove that if we take the union of two polygons that are congruent to P, the ratio of the perimeter and area of the resulting shape cannot be more than the ratio of the perimeter and area of P.

Problems in Physics

(see page 315)

M. 405. Measure the flow rate of a mixing faucet first when cold water flows from the tap, and then when hot water is flowing from the tap. Also measure the temperature of cold and hot water. Finally, measure the flow-rate of the faucet also for lukewarm water, and calculate what proportion of the lukewarm water is cold and hot water.

G. 745. In what states of matter does the material of a candle appear when the candle burns? **G. 746.** Birds sit in the closed cargo hold of a truck, carrying birds. When the vehicle horn honks loudly, the birds get frightened. Does the total weight of the truck and the birds increase, decrease or remain unchanged when the birds fly off? **G. 747.** Suppose that a one-atom thick layer is made from 1 kg gold. Estimate the number of football pitches that can be covered with this gold foil. **G. 748.** A 20-cm long upside down test-tube, whose bottom part is filled with water and whose top part contains air (a Cartesian diver), is placed into a tall graduated cylinder, filled with water. The top of the test-tube is a bit above the level of the water in the cylinder. The top of the graduated cylinder is covered with a sheet of rubber, and then this rubber sheet is pressed down such that the pressure inside the test-tube increases by 5 kPa. At this moment the "diver" starts to move downwards. *a*) What was the height of the air in the test-tube when the "diver" started to sink? *b*) What is the minimum height of the graduated cylinder, if the "diver" stays at the bottom of the cylinder even if the rubber sheet is taken away from the top of the cylinder?

P. 5326. An object is dropped from a tower of unknown height, and it falls freely. Air drag is negligible. *a*) Imagine we divide the height of the tower into two equal parts. Determine the ratio of the average speeds calculated for the two parts. *b*) How should the height of a 45 m high tower be split into two parts in order that the average speed calculated for the second part be four times as much as that calculated for the first part? **P. 5327.** How long would a day on the Earth be if we would “fall off” from the equator of the Earth, due to the rotation of the Earth, provided that the shape of the Earth was not changed? **P. 5328.** A light thin steel rod is clamped and held horizontally. A weight is attached to its free end such that it pulls that end to a 1 cm lower position than it was originally. If it is made to oscillate with small amplitude, what is the period of the oscillation? **P. 5329.** There is a piece of chalk at rest on a horizontal blackboard. The blackboard is suddenly pushed such that it gains a horizontal velocity of v_0, then it collides with a wall after a time of T and it suddenly stops. How long is the track of the chalk on the board if the coefficient of friction between the chalk and the board is μ? **P. 5330.** Imagine a spherical celestial body which is at liquid state. The internal gravitation causes hydrostatic pressure. Let the material of the celestial body be water and let its radius be $R = 25$ km. What is the hydrostatic pressure at the centre of the sphere? **P. 5331.** An old, popular toy is the potato rifle, which is made of a 12 cm long elder tube, whose cross sectional area is 0.3 cm^2. The two ends of the tube are plugged one after the other, each with a 1 cm long potato cylinder. One of the potato plugs acts as the projectile and the other as the piston. The potato cylinders seal the tube well. We know that at least 4 N force must be applied to move a potato cylinder (to overcome friction). In order to move the potato cylinder at a constant speed a force of 3.5 N is required. The latter force decreases to 0 in direct proportion to the length of the projectile in the barrel when the potato cylinder leaves the projectile. (The density of potato is 1.06 g/cm^3, and the external air pressure is 10^5 Pa.) *a*) What is the pressure of the air in the “loaded” rifle, which is sealed at its both ends? *b*) By means of a wooden stick, the potato plug is slowly pushed along the cylinder until the other potato cylinder, the projectile, suddenly pops out of the barrel. How much work do we have to do in order to “fire” a loaded rifle? *c*) At what speed does the potato projectile leave the barrel? **P. 5332.** A small ball of mass m and of charge $Q = 1$ μC hangs on a piece of insulating thread of length $L = 0.2$ m. At a distance of $2L$ below the suspension there is another small fixed ball of charge Q. *a*) How does the angle between the thread and the vertical depend on the mass m? *b*) What should the least value of m be in order to have a distance of L between the two balls? *c*) What should the maximum value of m be in order to have a distance of $3L$ between the two balls? **P. 5333.** Electric current flows in a long, straight cylindrical piece of wire of radius 2 cm. The magnitude of the magnetic flux density inside the wire at a distance of 1.5 cm from the symmetry axis of the wire is $2 \cdot 10^{-4}$ T. What is the magnitude of the magnetic flux density at a distance of 4 cm from the symmetry axis of the wire? **P. 5334.** Rudy, who loves physical experimentation, received an electronics kit for his birthday. He immediately assembled the circuit shown in the *figure.* The internal resistance of the current source of voltage $U = 30$ V is negligible, and the totally alike voltmeters and the totally alike ammeters are considered ideal. The magnitude of the resistances is $R = 50$ Ω. *a*) What are the readings on the meters? *b*) Then he swapped ammeter 1 for voltmeter 1 and he also swapped ammeter 2 for voltmeter 2. What are the readings on the meters now? *c*) Then he placed back all the meters to their original positions, and then he swapped ammeter 1 for voltmeter 2. What are the readings on the meters in this case? **P. 5335.** If a solid ball of diameter of 8 cm was made of pure ^{238}Pu, what would the temperature of the surface of this ball be when it is placed into a point in space at a temperature of -270 degrees Celsius, far away from everything. (Such an isotope is used to power “deep space” spacecrafts, travelling far from the Sun, in their radioisotope thermoelectric generators.) **P. 5336.** A supersonic fighter-plane flies 2 km above a fairly large, wide, flat field along a horizontal line. The sound of the fighter is heard at the same instant by three observers standing in the field, 14 km apart pairwise. The fighter is right above the head of one of the observers. What is the speed of the fighter?