(Ilyen izotópot használnak a Naptól távol haladó, „mélyűri" űrszondák energiaellátására a radioizotópos termoelektromos generátorokban.)

(*5 pont*) Közli: *Vass Miklós,* Budapest

P. 5336. Elég nagy kiterjedésű, széles, sík mező fölött 2 km magasan repül egy szuperszonikus vadászgép vízszintes irányban. A gép hangját a mezőn álló három, egymástól páronként 14 km-re lévő megfigyelő egyszerre hallja meg. A repülőgép éppen az egyik megfigyelő feje felett repül el. Mekkora a vadászgép sebessége?

(*6 pont*) Közli: *Vigh Máté,* Biatorbágy

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Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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Problems in Mathematics

New exercises for practice – competition C (see page 285): **Exercises up to grade 10: C. 1672.** Find all number pairs p, r such that p, r and $\frac{p+r}{p-r}$ are all positive and prime. **C. 1673.** A trapezium is divided into four triangles by its diagonals. The sum of the areas of the triangles lying on the bases of the trapezium make up $\frac{13}{18}$ of the area of the trapezium. Given that the length of one base is 5 cm, what may be the length of the other base? **Exercises for everyone: C. 1674.** Prove that there are infinitely many right-angled triangles in which the measures of the sides are positive integers, and the hypotenuse is one unit longer than one of the legs. (Proposed by *L. Németh,* Fonyód) **C. 1675.** Let D be an interior point of side AB in triangle ABC, and $\frac{AD}{DB}=\frac{m}{n}<\frac{1}{2}$, where m, n are positive integers. A point E, different from D, is marked on the circumference of the triangle such that line DE divides the area of the triangle in a $1:2$ ratio. Depending on the numbers m and n, on which side of the triangle will point E lie, and in what ratio will it divide that side? **C. 1676.** Show that $2019^{2021}+2021^{2019}$ is divisible by 4040. Determine whether the following generalization of the problem is also true: if a and b are consecutive odd positive integers then a^b+b^a is divisible by $a+b$. **Exercises upwards of grade 11: C. 1677.** Solve the equation $\left|2\cdot\log_2\sqrt{x^2-x}+3+\frac{1}{\log_4\sqrt{x^2-x}}\right|=2$ over the set of real numbers. **C. 1678.** The length of each edge of a square-based regular pyramid is a. Connect the centres of the faces of the pyramid in every possible way. Prove that one can always construct a triangle using any three such line segments.

New exercises – competition B (see page 286): **B. 5174.** Prove that $(2n)!\leqslant(n^2+n)^n$ for all positive integers n. (*3 points*) (Proposed by *M. Szalai,* Szeged) **B. 5175.** In a triangle ABC, $AC=BC$, D is an interior point of side AC, and K is the centre of the circle ABD. Show that quadrilateral $BCDK$ is cyclic. (*3 points*) **B. 5176.** The first n positive integers need to be written on the circumference of a circle (each number exactly once), so that the sums of all sets of three adjacent numbers should form exactly two different values. Find all possible values of n. (*4 points*) (*Scottish competition problem*) **B. 5177.** In a right-angled triangle ABC, line segment CD is the altitude drawn to the hypotenuse. The circle k of diameter CD intersects the legs AC and BC again at points E and F, respectively. The tangent drawn to circle k at point E intersects the line of leg BC at point P, and the hypotenuse AB at point M. The tangent drawn to circle k at point F intersects the line of leg AC at point Q, and the hypotenuse AB at point N. Prove that $4 \cdot MN^2 = PE^2 + QF^2 + 2 \cdot EF^2$. (*5 points*) **B. 5178.** Let x be a positive real number. Show that $\sqrt{6x+9} + \sqrt{16x+64} \leqslant \left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{8}{\sqrt{x}}\right)$. (*4 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5179.** Is there a set H of integers with the following property: every nonzero integer can be represented in infinitely many ways as a sum of some distinct elements of H, but 0 cannot be represented at all? (*6 points*) **B. 5180.** The radius of the circumscribed circle of a regular heptagon $ABCDEFG$ is r. Prove that the circle of radius $2r$ centred at A passes through the orthocentre of triangle BCE. (*5 points*) **B. 5181.** Given eight points on the plane, no three of which are collinear and no five of which are concyclic, what is the maximum possible number of circles that pass through four of the points each? (*6 points*) (Proposed by *A. Imolay,* Budapest)

New problems – competition A (see page 288): **A. 800.** In a finite, simple, connected graph G we play the following game: initially we color all the vertices with a different color. In each step we choose a vertex randomly (with uniform distribution), and then choose one of its neighbors randomly (also with uniform distribution), and color it to the the same color as the originally chosen vertex (if the two chosen vertices already have the same color, we do nothing). The game ends when all the vertices have the same color. Knowing graph G find the probability for each vertex that the game ends with all vertices having the same color as the chosen vertex. (Submitted by *Dávid Matolcsi,* Budapest) **A. 801.** For which values of positive integer m is it possible to find polynomials $p, q \in \mathbb{C}[x]$ with degrees at least two such that $x(x+1)\cdots(x+m-1) = p\big(q(x)\big)$? (Submitted by *Navid Safaei,* Tehran) **A. 802.** Let P be a given regular 100-gon. Prove that if we take the union of two polygons that are congruent to P, the ratio of the perimeter and area of the resulting shape cannot be more than the ratio of the perimeter and area of P.

Problems in Physics

(see page 315)

M. 405. Measure the flow rate of a mixing faucet first when cold water flows from the tap, and then when hot water is flowing from the tap. Also measure the temperature of cold and hot water. Finally, measure the flow-rate of the faucet also for lukewarm water, and calculate what proportion of the lukewarm water is cold and hot water.

G. 745. In what states of matter does the material of a candle appear when the candle burns? **G. 746.** Birds sit in the closed cargo hold of a truck, carrying birds. When the vehicle horn honks loudly, the birds get frightened. Does the total weight of the truck and the birds increase, decrease or remain unchanged when the birds fly off? **G. 747.** Suppose that a one-atom thick layer is made from 1 kg gold. Estimate the number of football pitches that can be covered with this gold foil. **G. 748.** A 20-cm long upside down test-tube, whose bottom part is filled with water and whose top part contains air (a Cartesian diver), is placed into a tall graduated cylinder, filled with water. The top of the test-tube is a bit above the level of the water in the cylinder. The top of the graduated cylinder is covered with a sheet of rubber, and then this rubber sheet is pressed down such that the pressure inside the test-tube increases by 5 kPa. At this moment the "diver" starts to move downwards. *a*) What was the height of the air in the test-tube when the "diver" started to sink? *b*) What is the minimum height of the graduated cylinder, if the "diver" stays at the bottom of the cylinder even if the rubber sheet is taken away from the top of the cylinder?

P. 5326. An object is dropped from a tower of unknown height, and it falls freely. Air drag is negligible. *a*) Imagine we divide the height of the tower into two equal parts. Determine the ratio of the average speeds calculated for the two parts. *b*) How should