

denote the expected number of rounds that was needed to get rid of all the coins. Prove that there exists $c > 0$ for which the following inequality holds for all positive integers n : $c\left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right) < k_n < 1 + c\left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right)$. **A. 797.** For a given quadrilateral $A_1A_2B_1B_2$ point P is called *phenomenal*, if line segments A_1A_2 and B_1B_2 subtend the same angle at point P (i.e. triangles PA_1A_2 and PB_1B_2 which can be also degenerate have equal inner angles at point P disregarding orientation). Three non-collinear points, A_1 , A_2 and B_1 are given on the plane. Prove that it is possible to find a disc on the plane such that for every point B_2 on the disc quadrilateral $A_1A_2B_1B_2$ is convex for which it is possible to construct seven distinct phenomenal points only using a right ruler. With a right ruler the following two steps are allowed: *i*) given two points it is possible to draw the straight line connecting them; *ii*) given a point and a straight line, it is possible to draw the straight line passing through the given point which is perpendicular to the given line. (Proposed by *Á. Bán-Szabó*, Budapest)

Problems in Physics

(see page 249)

M. 404. Measure the period (T_1) of a thin-walled ball, which is suspended as shown in the *figure* and is displaced a little perpendicularly to the plane of the threads. Then turn a bit the initially stationary ball, about its vertical axis and measure the period of the torsional vibration (T_2). From the measured value calculate the ratio of $\frac{T_1}{T_2}$.

G. 741. Suppose that *Elon Musk* —the multibillionaire known from his whimsical ideas— wants to determine the number of geosynchronous satellites such that he sends a counting satellite next to the path of the geosynchronous satellites. This satellite does not move west to east, but oppositely from east to west. How long does it take for this satellite to count all the satellites, which seem to be at rest with respect to the Earth?

G. 742. The friction between a 20 kg crate and a straight inclined plane is so big that the crate does not slide down by itself. This crate can be pulled up whilst 3.0 kJ work is done and it can be moved down with 1.0 kJ work. (The pulling force is parallel to the plane of the slope, and the motion of the crate is very slow.) What is the height of the slope?

G. 743. A fully packed wall cabinet has a shape of a cuboid of width $a = 40$ cm, height $b = 75$ cm. The (total) mass of the cabinet is 40 kg, and its centre of mass is at its geometric centre. The cabinet is mounted to the wall by means of two screws inserted into wall plugs. The screws are at the two top vertices of the cuboid next to the wall (the *figure* shows a side view of the cabinet, point P is the overlapping position of the two screws). The cabinet touches the wall only along one of its edges at its bottom base. At least what magnitude of pulling force must the fasteners separately withstand, so that the screws are not torn out of the wall? (Neglect friction at the wall.)

G. 744. The circuit shown in the *figure* consists of four alike resistors each of resistance 10Ω and a battery. *a*) What is the terminal voltage of the battery if the power dissipation at the resistor which dissipates the greatest thermal energy is 360 W? *b*) What is the dissipated power at the other resistors?

P. 5315. A cyclist is travelling at a constant speed of 9 km/h on a level road, and then in 20 seconds he speeds up uniformly to the speed of 18 km/h. What is the acceleration of a point on the rim of the wheel right after the accelerating period ended? The diameter of the wheel is 72 cm. How much distance was covered, and how many times did the wheel turn in the accelerating period of the motion?

P. 5316. An $m_2 = 1$ kg disc sliding at a speed of $v_0 = 5$ m/s collides head on with another disc of mass m_1 resting on the horizontal rough tabletop. The coefficient of kinetic friction between the table and

the discs of masses m_1 and m_2 are $\mu_1 = 0.1$ and $\mu_2 = 0.25$, respectively. *a)* What is the mass of the initially stationary disc, if after the totally elastic collision the two discs stop at the same instant? *b)* How far are they from each other when they stopped? *c)* After the collision how much time elapses until the discs stop? **P. 5317.** The principal axis of a school telescope is pointing exactly towards the centre of the Moon at full moon. The image of the Moon in the stationary telescope, which is in the school yard, totally fills the field of view. How much time elapses between the appearance and the disappearance of the full moon? **P. 5318.** According to some ideas, the hypothetical particles of dark matter (blacktons) move in intergalactic space such that a force opposite to their speed is exerted on them. It is not known yet, how this force depends on the speed of the particle, but it is known that the ratio of the restoring forces of two regions in which the density is different is the same for any speed. Some blackton particles with initial velocity of v_0 stop after covering a distance of 3 light years, in some low-density region, whereas they stop in a distance of 2 light years in some greater-density region. How much distance do blacktons with initial speed of v_0 cover if they first travel in the greater-density region of width 1.5 light years, and then they enter into the lower-density region? **P. 5319.** A uniform-density thin stick of length ℓ and of mass m is made slide along a horizontal surface. At a certain moment the velocity of one end of the stick is \mathbf{v}_1 , whilst that of the other end is \mathbf{v}_2 . At this moment what is the *a)* linear momentum of the stick; *b)* angular momentum of the stick with respect to its centre of mass; *c)* its total kinetic energy? **P. 5320.** Two nails at a distance of L are sticking out of a vertical wall at the same height. A piece of rope is placed to the nails such that it sags to a depth of H . Estimate the length of the rope if it is known that *a)* $H \ll L$; *b)* $L \ll H$. Friction is negligible everywhere. **P. 5321.** Pistons with different masses, which can move frictionlessly, confine samples of oxygen and helium gas into vertical cylinders sealed at their bottoms. The volume of the gases is equal and they are at the same temperature. The two samples of gas are heated slowly to the same temperature. During the heating process the change in the internal energy of the oxygen gas is 2.5 times greater than that of the helium gas, and its work done during the expansion is 220 J greater than the work done by the sample of helium gas. *a)* Determine the ratio of the pressure of the gases at their initial state. *b)* How much heat was absorbed by the oxygen and by the helium during the heating process? **P. 5322.** The size of the sensor of a digital camera is $23.5 \text{ mm} \times 15.6 \text{ mm}$, and this sensor can capture 6045×4003 pixels. A side photo of a motorboat travelling at a speed of 40 km/h at a distance of 20 m is taken with this camera. What should the exposure time of the camera, which has an objective lens whose focal length is 35 mm, be in order not to gain a blurred image of the motorboat? **P. 5323.** A current $I = 40 \text{ mA}$ flows into a system of resistors shown in the *figure* at point *A* and flows out of it at point *B*. *a)* What is the value of the current flowing through each resistor? *b)* What is the dissipated power at each resistor? *c)* What is the resistance of a single resistor with which we could replace the whole system of resistors? **P. 5324.** Some rock mined in one shift in a uranium mine near Kinshasa in Central Africa contained 10 tons of pitchblende (U_3O_8). Estimate the amount of radium in the rock. **P. 5325.** A freezer has been operated in a store room for a long time. The temperature inside the freezer is -20°C , the temperature in the store-room is 25°C , and everywhere else in the flat the temperature is 20°C . After a long time of operation what will the temperature of the store room be if another alike freezer is placed into the store-room? Assume that —apart from the store room— the temperature of the flat does not change. Consider the freezers to be ideal Carnot refrigerators, whose thermostats are adjusted to maintain the inside temperature of -20°C .