Feltehetjük, hogy a lakás hőmérséklete a kamrán kívül nem változik. A hűtőládákat tekintsük ideális Carnot-gépeknek, amelyek termosztátja úgy van beállítva, hogy belül fenntartja a −20 °C-os hőmérsékletet.

(*6 pont*) Közli: *Vigh Máté,* Biatorbágy

Áprilisi pótfeladat.* Egy függőleges falú medence a csap kinyitása után T idő múlva telik meg. Ezt a vízmennyiséget a lefolyónyílás megnyitása után $2T$ idő alatt lehet leereszteni. Mennyi idő alatt telik meg a medence, ha nyitott lefolyónyílás mellett szeretnénk a medencét a csap megnyitásával feltölteni?

Közli: *Radnai Márton,* Budapest

❄

Beküldési határidő: 2021. május 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

❄

kérdőív diákok részére

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kérdőív nem diákok részére

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Problems in Mathematics

New exercises for practice – competition C (see page 223): **Exercises up to grade 10: C. 1665.** Each letter of the word *KÖMAL* denotes a digit in decimal notation. Given the equalities below, determine the value of the five-digit number $\overline{K\ddot{O}MAL}$. (1) $M+\ddot{O}+L=\overline{KA}$, (2) $\ddot{O}+L=\overline{KK}$, (3) $K+\ddot{O}+M=10$, (4) $A\cdot L=42$. **C. 1666.** In an acute-angled triangle ABC, let K and D, respectively, be the intersections of the interior angle bisector drawn from point A with the interior angle bisector drawn from B, and with side BC. The perpendicular drawn to angle bisector AD at point K intersects side AB at point E. F is the foot of the perpendicular drawn from point E to BC. T is the foot of the perpendicular drawn from point D to line AB. Prove that T lies on the circumscribed circle of triangle KEF. **Exercises for everyone: C. 1667.** Let $A=(-1)^1+(-1)^2+(-1)^3+\dots+(-1)^{2021}$, $B=(-2)^1+(-2)^2+(-2)^3+\dots+(-2)^{2021}$

*A pótfeladat megoldása beküldhető e-mailben a `szerk@komal.hu` címre, de nem számít bele a pontversenybe.

and $C=(-3)^1+(-3)^2+(-3)^3+\dots+(-3)^{2021}$. Determine the last digit of the number $B+C-A$. **C. 1668.** The midpoints of sides AB, BC, CD, DA of a parallelogram $ABCD$ are E, F, G, H, respectively. The lines AF and AG intersect diagonal BD at points K and L, respectively. Show that the sum of the areas of triangles EFK and GHL equals the area of triangle EKL. **C. 1669.** Let N be $\overline{abc}$ a three-digit number in decimal notation. The value of a number $M=\overline{abc}$ represented in some non-decimal notation is $2N$. Determine the number N. **Exercises upwards of grade 11: C. 1670.** Given that a and b are integers such that $3a-2b$ is divisible by 13, prove that $4a+19b$ and $38a+57b$ are also divisible by 13. **C. 1671.** Line segments AE, BF, CG, DH are perpendicular to the plane S of parallelogram $ABCD$, in the same half space formed by plane S. T and t denote the areas of quadrilaterals $CGEA$ and $DHFB$, respectively. Prove that if $\frac{T}{t}=\frac{AC}{BD}$, then the points E, F, G, H are coplanar.

New exercises – competition B (see page 224): **B. 5166.** Are there prime numbers p, r greater than 3 such that the sum of the digits of $2p^2+7r^2+2021$ should be a perfect square? (*3 points*) **B. 5167.** Consider two circles in the plane that have common interior tangents. Show that the circle passing through the points of contact of the internal tangents bisects the line segment connecting the centres of the two original circles. (*3 points*) (Proposed by *the class 8C of Fazekas Mihály Primary and Secondary Grammar School of Budapest*) **B. 5168.** Each of the integers 1 to 100 is written on a piece of paper. 16 pieces of paper are selected out of the 100 pieces. Is it certain that there will always be four pieces of paper among the selected ones such that the sum of the numbers on two of them equals the sum of the numbers on the other two? (*6 points*) **B. 5169.** Find the real solutions of the equation $\sqrt[3]{2x+11}+\sqrt[3]{3x+4}=\sqrt[3]{x+9}+\sqrt[3]{4x+6}$. (*5 points*) (Proposed by *M. Szalai,* Szeged) **B. 5170.** Let α and β be acute angles such that $\sin^2\alpha+\sin^2\beta=\sin(\alpha+\beta)$. Prove that $\alpha+\beta=90^\circ$. (*4 points*) **B. 5171.** Let $OLMN$ be a tetrahedron, and the vertices A, B and C of another tetrahedron $OABC$ lie on the rays OL, OM and ON, respectively. The centre of the inscribed circle of triangle LMN coincides with the centroid of triangle ABC. Show that the volume of tetrahedron $OLMN$ is greater than or equal to the volume of tetrahedron $OABC$. On what condition will the volumes of the two tetrahedra be equal? (*5 points*) (*From the British qualifying competition for the olympiad, 1980*) **B. 5172.** Six regular dice are placed in a cup, and rolled simultaneously. Those dice that do not show a 6 are returned to the cup, and rolled again. If there are dice that still not show a 6, those dice are rolled a third time. The procedure is repeated until every dice shows a 6. What is the probability that exactly six rolls are needed? (*6 points*) **B. 5173.** The orthocentre of an acute-angled triangle ABC is H, and the centre of the circumscribed circle is O. Let D and E denote interior points on the line segments AB and AC, respectively. The orthocentre and circumcentre of triangle ADE are H' and O', respectively. Show that lines HH' and OO' are parallel if and only if $BD=CE$. (*6 points*) (Proposed by *Á. Bán-Szabó,* Budapest)

New problems – competition A (see page 226): **A. 797.** We call a system of non-empty sets H *entwined,* if for every disjoint pair of sets A and B in H there exists $b\in B$ such that $A\cup\{b\}$ is in H or there exists $a\in A$ such that $B\cup\{a\}$ is in H. Let H be an entwined system of sets containing the following n one-element sets: $\{1\},\{2\},\dots,\{n\}$. Prove that if $n>k(k+1)/2$, then H contains a set with at least $k+1$ elements, and this is sharp for every k, i.e. if $n=k(k+1)$, it is possible that every set in H have at most k elements. **A. 798.** Let $0<p<1$ be given. Initially we have n coins, all of which has probability p of landing on heads, and probability $1-p$ landing on tails (the results of the tosses are independent from each other). In each round we toss our coins and remove those that result in heads. We keep repeating this until all our coins are removed. Let k_n

denote the expected number of rounds that was needed to get rid of all the coins. Prove that there exists $c>0$ for which the following inequality holds for all positive integers n: $c\Big(1+\frac{1}{2}+\dots+\frac{1}{n}\Big)<k_n<1+c\Big(1+\frac{1}{2}+\dots+\frac{1}{n}\Big)$. **A. 797.** For a given quadrilateral $A_1A_2B_1B_2$ point P is called *phenomenal,* if line segments A_1A_2 and B_1B_2 subtend the same angle at point P (i.e. triangles PA_1A_2 and PB_1B_2 which can be also also degenerate have equal inner angles at point P disregarding orientation). Three non-collinear points, A_1, A_2 and B_1 are given on the plane. Prove that it is possible to find a disc on the plane such that for every point B_2 on the disc quadrilateral $A_1A_2B_1B_2$ is convex for which it is possible to construct seven distinct phenomenal points only using a right ruler. With a right ruler the following two steps are allowed: *i*) given two points it is possible to draw the straight line connecting them; *ii*) given a point and a straight line, it is possible to draw the straight line passing through the given point which is perpendicular to the given line. (Proposed by *Á. Bán-Szabó,* Budapest)

Problems in Physics

(see page 249)

M. 404. Measure the period (T_1) of a thin-walled ball, which is suspended as shown in the *figure* and is displaced a little perpendicularly to the plane of the threads. Then turn a bit the initially stationary ball, about its vertical axis and measure the period of the torsional vibration (T_2). From the measured value calculate the ratio of $\frac{T_1}{T_2}$.

G. 741. Suppose that *Elon Musk* —the multibillionaire known from his whimsical ideas— wants to determine the number of geosynchronous satellites such that he sends a counting satellite next to the path of the geosynchronous satellites. This satellite does not move west to east, but oppositely from east to west. How long does it take for this satellite to count all the satellites, which seem to be at rest with respect to the Earth? **G. 742.** The friction between a 20 kg crate and a straight inclined plane is so big that the crate does not slide down by itself. This crate can be pulled up whilst 3.0 kJ work is done and it can be moved down with 1.0 kJ work. (The pulling force is parallel to the plane of the slope, and the motion of the crate is very slow.) What is the height of the slope? **G. 743.** A fully packed wall cabinet has a shape of a cuboid of width $a=40$ cm, height $b=75$ cm. The (total) mass of the cabinet is 40 kg, and its centre of mass is at its geometric centre. The cabinet is mounted to the wall by means of two screws inserted into wall plugs. The screws are at the two top vertices of the cuboid next to the wall (the *figure* shows a side view of the cabinet, point P is the overlapping position of the two screws). The cabinet touches the wall only along one of its edges at its bottom base. At least what magnitude of pulling force must the fasteners separately withstand, so that the screws are not torn out of the wall? (Neglect friction at the wall.) **G. 744.** The circuit shown in the *figure* consists of four alike resistors each of resistance 10 Ω and a battery. *a*) What is the terminal voltage of the battery if the power dissipation at the resistor which dissipates the greatest thermal energy is 360 W? *b*) What is the dissipated power at the other resistors?

P. 5315. A cyclist is travelling at a constant speed of 9 km/h on a level road, and then in 20 seconds he speeds up uniformly to the speed of 18 km/h. What is the acceleration of a point on the rim of the wheel right after the accelerating period ended? The diameter of the wheel is 72 cm. How much distance was covered, and how many times did the wheel turn in the accelerating period of the motion? **P. 5316.** An $m_2=1$ kg disc sliding at a speed of $v_0=5$ m/s collides head on with another disc of mass m_1 resting on the horizontal rough tabletop. The coefficient of kinetic friction between the table and