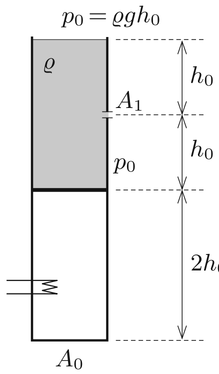

P. 5314. Egy függőleges, jól hőszigetelt, felül nyitott, A_0 keresztmetszetű hengert két részre oszt egy szintén hőszigetelő anyagból készült, elhanyagolható vastagságú és tömegű dugattyú. A dugattyú alatti $2h_0$ magasságú térrészben levegő, míg felette $2h_0$ magasságban ϱ sűrűségű folyadék található. A dugattyú felett h_0 magasságban a folyamat kezdetén megnyitunk egy apró, A_1 keresztmetszetű nyílást, melyen a folyadék elkezd kifolyni a hengerből. A számítások során a légköri nyomás értékét vegyük $p_0 = \varrho g h_0$-nak.

a) Hogyan változtassuk az elzárt levegőt melegítő fűtőszál teljesítményét az idő függvényében, hogy a folyadék állandó sebességgel folyjon ki a nyíláson?

b) Mennyi ideig tudjuk biztosítani a folyadék állandó sebességű kiáramlását, és ezen időpont után ez már miért nem oldható meg a fűtőszállal?

(*6 pont*) Közli: *Olosz Balázs,* Budapest

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Problems in Mathematics

New exercises for practice – competition K (see page 161): **K. 689.** In the 6th, 7th, 8th and 9th games of the season, a basketball player scored 23, 14, 11 and 20 points, respectively. His points average was higher after the 9th game than after the 5th game. With the 10th game, his average rose above 18. What is the lowest possible number of points that he may have scored in the 10th game? **K. 690.** Having a positive integer in mind, Peti formulated twenty-three statements about it. Two consecutive statements are false, but the rest of them are true: 1. It is divisible by 2. 2. It is divisible by 3. 3. It is divisible by 4. … 23. It is divisible by 24. Peti was thinking about the smallest such number. What is his number? **K. 691.** $ABCDEFGH$ is a regular octagon and its sides are 2 units long. Squares $BCIM$ and $FGKL$ are drawn on sides BC and GF, on the inside. What is the area of the rectangle bounded by lines AH, KL, ED and IM? **K. 692.** A 6×6 square is dissected into lattice rectangles. What is the largest possible number of noncongruent rectangles obtained? Give an example. **K. 693.** Quadrilateral $ABCD$ has an inscribed circle centred at O. Show that the sum of $\angle DOC$ and $\angle BOA$ is 180°.

New exercises for practice – competition C (see page 161): **Exercises up to grade 10: C. 1658.** A circular disc is divided into six congruent sectors. A circle is inscribed in each sector. The circle touches the arc of the sector as well as the two radii. What fraction of the area of the large circle is covered by the six smaller circles? **C. 1659.** Ray a starts from point A of a line segment AB, and encloses an angle $0^\circ < \alpha < 90^\circ$

with it. Ray b starts from point B, and encloses an angle $0^\circ < \beta < 90^\circ$ with the line segment AB. The two rays lie in two different half planes of a plane containing line AB. The circle of diameter AB intersects a again at A_1, and b at B_1. The circle of diameter A_1B_1 intersects the line containing a again at A_2, and the line containing b at B_2. What is the relationship between α and β if A_1B_1 and A_2B_2 are perpendicular? **Exercises for everyone: C. 1660.** The positive integers 1 to 61^2 are written in the fields of a 61×61 chessboard, starting from the top left corner and proceeding along each row in succession. Then some changes are made as follows. In the first move, the sign of each number is changed to negative. In the second move, the signs of all even numbers are changed. In the third move, the sign of every multiple of 3 is changed, and so on, while the moves are meaningful. When all this is completed, how many 1×2 rectangles will there be on the chessboard in which the sum of the numbers is negative? **C. 1661.** In a lottery game, the player bets 5 numbers out of the positive integers 1 to 90. Otto Lotter insists on increase, and he always keeps the following rules when making his bets: he marks 5 numbers such that every digit may only occur once, and if the five numbers are listed in increasing order, the digits must be ascending, too. For example, 1, 2, 3, 46, 78. How many suitable selections of five numbers are there? (Proposed by *Berkó Erzsébet,* Szolnok) **C. 1662.** For what values of the real parameter $a > 0$ will the equation $x^2 + a = \sqrt{x - a}$ have exactly one solution in the set of real numbers? What is the solution of the equation in that case? **Exercises upwards of grade 11: C. 1663.** The circles k_1 and k_2 touch each other externally at point E. Lines f and g pass through point E. One of the common external tangents touches the circle k_1 and k_2 at points C and D, respectively. Line h is obtained by dropping perpendiculars from point C onto lines f and g, and connecting the feet of the perpendiculars. Line m is obtained in the same way, with perpendiculars from point D. Prove that h and m are perpendicular to each other. **C. 1664.** Each of the diagonals AD, BE and CF of a convex hexagon $ABCDEF$ halves the area of the hexagon. Prove that these diagonals are concurrent.

New exercises – competition B (see page 163): **B. 5158.** Let A, B, C and D be points in the plane such that $AB < CB$ and $CD < AD$. Prove that the line segments AB and CD do not intersect each other. (*3 points*) **B. 5159.** Solve the equation $\left[\frac{2020-x}{x-1}\right] + \left[\frac{2021+x}{x+1}\right] = 82$ over the set of integers, where $[c]$ denotes the greatest integer not greater than c. (*4 points*) (Proposed by *Zs. M. Tatár,* Esztergom) **B. 5160.** What may be the value of $x+y+z$ if $\sqrt{x-1} + 2\sqrt{y-4} + 3\sqrt{z-9} = \frac{x+y+z}{2}$? (*3 points*) (Proposed by *M. Szalai,* Szeged) **B. 5161.** 800 L-tetrominoes are laid on a 100×100 chesssboard. Prove that it is possible to place another L-tetromino on the board. The tetrominoes do not overlap, and each of them covers exactly 4 fields of the chessboard. An L-tetromino is defined as the shape shown in the *figure,* including any rotated or reflected images. (*6 points*) **B. 5162.** The sides of triangle ABC are 9, 10 and 17 units long. What is the area of the triangle formed by the exterior angle bisectors of triangle ABC? (*5 points*) (Proposed by *Zs. M. Tatár,* Esztergom) **B. 5163.** Triangle ABC is right-angled at C. The right angle is divided $2:1$ by a line such that the smaller part is closer to the shorter leg. This line intersects hypotenuse AB at T, and the circumscribed circle at D. What are the measures of the acute angles of the triangle if the feet of the perpendiculars dropped from D onto the lines of the legs are collinear with T? (*4 points*) **B. 5164.** Two players continue playing rock-paper-scissors games until one of them wins the third time. Assume that each player in each game selects rock, paper or scissors at random (independently of each other and of previous games), with probabilities of $\frac{1}{3} : \frac{1}{3} : \frac{1}{3}$. Determine the expected value of the number of games needed. (*5 points*) **B. 5165.** Given a positive integer k, is there a function $f : \mathbb{N} \to \mathbb{N}$, such that $f(x) + f\big(f(x)\big) = x + k$ for all $x \in \mathbb{N}$? (*6 points*) (Proposed by *M. Lovas,* Budapest)

New problems – competition A (see page 164): **A. 795.** The following game is played with a group of n people: $n+1$ hats are numbered from 1 to $n+1$. The people are blindfolded, and each of them is getting one of the $n+1$ hats on his head (the remaining hat is hidden). Now a line is formed from the n people, and their eyes are uncovered: each of them can see the numbers on the hats of the people standing in front of him. Now starting from the last person (who can see all the other players) the players take turns to guess the number of the hat on their head, but no two players can guess the same number (each player hears all the guesses from the other players). What is the highest number of guaranteed correct guesses, if the n people can discuss a common strategy after learning about the game? (Submitted by *Viktor Kiss,* Budapest) **A. 796.** Let $ABCD$ be a cyclic quadrilateral. Let lines AB and CD intersect in P, and lines BC and DA intersect in Q. The feet of the perpendiculars from P to BC and DA are K and L, and the feet of the perpendiculars from Q to AB and CD are M and N. The midpoint of diagonal AC is F. Prove that the circumcircles of triangles FKN and FLM, and the line PQ are concurrent. (Based on a problem by *Ádám Péter Balogh,* Szeged)

Problems in Physics

(see page 185)

M. 403. For some commercially available granular matter (e.g. lentils, rice, egg barley, etc.) determine by measurement what percentage of their storage volume is air.

G. 737. To how many equal parts should a piece of wire of resistance 100 Ω be cut, in order to gain an equivalent resistance of 1 Ω when the pieces are connected in parallel? **G. 738.** When an earthquake occurs several waves are generated from the centre of the quake. The so called primary waves (P-waves) are the fastest, in our case their propagation speed is 5 km/s. The secondary waves (S-waves) are slower, they travel at a speed of 3 km/h. Two seismic observatories are at a distance of 75 km from each other. In one of them 6 seconds were measured between the detection of the P and S waves, whilst in the other 8 seconds elapsed. At what maximum depth could the centre of the earthquake be? **G. 739.** Improperly positioned loads can tip over the fork-lift trunk. Therefore a so-called load capacity chart is attached to the truck (see the *figure*). Using the chart determine the horizontal distance from the heel of the fork to the axle of the front wheel of the truck, and the horizontal distance between the heel of the fork and the centre of mass of the truck of mass 1200 kg. **G. 740.** The asymmetrical U-shaped tube shown in the *figure* has a cross-sectional area of 2 cm^2, and it is filled with mercury. The top of the tube at the left is sealed, thus in this arm, there will be an air column of height 10 cm, at a pressure of 1 atm. How many cm^3 of mercury should be filled slowly and carefully into the right arm, in order to decrease the level of mercury in the right arm to half of the original height?

P. 5305. Mary is sitting in a rotating merry-go-round, admiring the gingerbread she has just received from Ian. The radius of the circular path of the gingerbread is $R = 5$ m, its period is $T_0 = 5$ s and the plane of its path is at a height of $H = 3.2$ m. Mary is so careless that she accidentally drops her gift. How far is Mary's hand, which she does not move, from the gingerbread at the moment when the gingerbread touches the ground? Neglect the size of the gingerbread and air resistance. **P. 5306.** A bicycle-like vehicle with only one wheel, called unicycle, is at rest in an unstable position. We would like to move further with it. To do this we have to speed up first, then we need to move uniformly and then we have to slow down and leave the unicycle in the original unstable position. How do we need to pedal in order not to fall from the unicycle? **P. 5307.** The pumping rate of