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Problems in Mathematics

New exercises for practice – competition K (see page 96): **K. 684.** *a*) Sophie and Bertie are playing a game that involves breaking a bar of chocolate into pieces. The chocolate bar consists of 10×5 squares. They take turns in splitting the chocolate along the dividing lines. The player who first breaks off a single square will lose the game. In each move, they are only allowed to touch and split one piece. Sophie starts the game. Can she make sure that she will win, whatever Bertie's moves are? *b*) After the first game, Bertie wants to strike back. He wants Sophie to start again, but with a different rule: the player who first breaks off a single square will win the game. Can Sophie make sure that she will win again? **K. 685.** Steve went picking mushrooms. Since he is becoming better and better at spotting mushrooms, this time he found 62 penny bun mushrooms. The average number in his previous mushroom picking trips had been 30, which was thus increased to 32. How many penny bun mushrooms should he have found in order to increase the mean to 33? **K. 686.** Each of the integers 1 to 100 is written on a separate piece of paper. 20 pieces of paper are drawn at random from the 100 pieces. Show that it is always possible to select four out of the 20 such that the sum of two numbers equals the sum of the other two. **K. 687.** There are some toy robots waiting on one side of a street. In each move, it is allowed to instruct exactly four robots to cross the street. For what number of robots is it possible to make all the robots end up on the other side? **K. 688.** *a*) Is it possible to form pairs out of the numbers $1,2,3,4,\dots,23,24$, so that the sum of each pair should be a perfect square? *b*) Is it possible to form pairs out of the numbers $1,2,3,4,\dots,21,22$ so that the sum of each pair should be a perfect square?

New exercises for practice – competition C (see page 97): **Exercises up to grade 10: C. 1651.** The terms of a number sequence are generated as follows: the first term is 895, and the following term is always obtained by multiplying the sum of the digits of the previous term by 61. Determine the 2021st term of the sequence, and the sum of the first 2021 terms. **C. 1652.** The shorter leg of each of two right-angled triangles has unit length. In each triangle, the right-angled vertex is at a unit distance from a point dividing the hypotenuse in a $2:1$ ratio: in one case it is the point closer to the right-angled vertex, and in the other case it is the point farther away. Prove that it is possible to select three out of the non-unit sides of the two triangles such that the three lengths form a right-angled triangle. **Exercises for everyone: C. 1653.** How many solutions does the inequality $|x|+|y|<2021$ have over the set of pairs of integers? **C. 1654.** Find the radius of each circle that is tangent to the graphs of the functions $f(x)=\frac{3x-6}{4}$ and $g(x)=\frac{28-4x}{3}$, and also touches the x-axis. **C. 1655.** Solve the equation $2(x+y-1831)^2=(2x-1802)(2y-1860)$ over the set of pairs of real numbers. **Exercises upwards of grade 11: C. 1656.** Three consecutive terms of an arithmetic sequence are prime numbers greater than 3. Show that the common difference of the sequence is divisible by 3. (Proposed by *L. Németh,* Fonyód) **C. 1657.** BCD and CAE are regular triangles drawn on legs BC and CA of a right-angled triangle ABC, on the outside. Prove that the midpoints of the line segments AB, CD and CE also form a regular triangle.

New exercises – competition B (see page 98): **B. 5150.** Prove that there are only a finite number of positive integers that cannot be obtained by adding one of the