complement in $S$, exactly one of these sets is contained in $U$.) See also problem **N. 35.**[*] from the May issue of 1994 (in Hungarian). **A. 792.** Let $p \geqslant 3$ be a prime number and $0 \leqslant r \leqslant p-3$. Let $x_1, x_2, \ldots, x_{p-1+r}$ be integer numbers satisfying $\sum_{j=1}^{p-1+r} x_j^k \equiv r \pmod{p}$ for all $1 \leqslant k \leqslant p-2$. What are the possible remainders of numbers $x_1, x_2, \ldots, x_{p-1+r}$ modulo $p$? (Submitted by *Dávid Matolcsi,* Budapest)

## Problems in Physics

(see page 57)

**M. 401.** Make a physical pendulum of mass $m$ and of length $\ell$, from a uniform-density thin wooden slat which is pivoted at one of the ends. (The values of $m$ and $\ell$ can be chosen arbitrary, but should be kept constant during the measurement.) *a*) Measure the period of the pendulum $T_0$ after it is displaced a bit. Then change the position of the pivot by positioning it at a distance of $d$ from one of the ends of the rod, and attach a point-like object of mass $M$, for example a small piece of plasticine, to the other end. If the mass of the plasticine is chosen carefully, then the period of this pendulum is the same as the original period $T_0$. *b*) Measure how the ratio of the masses $M/m$ depends on the ratio of the distances $d/\ell$.

**G. 729.** When melted lard is left to cool down in a pot, it can be observed clearly that the surface of the lard is similar to a crater, along the rim a regular flange is formed. Why? **G. 730.** In a bicycle race the first and the second riders are cycling at a constant speed of $v_0 = 50$ km/h. The first rider is 100 m ahead of the second. At a certain moment —close to the finish— the third cyclist begins to speed up and overtakes the second rider at a speed of $v_1 = 55$ km/h, and he is able to maintain this speed. How far is the finish from the point where the overtaking occurred if the first cyclist wins the race? **G. 731.** In a suburban area, where the the speed limit is 30 km/h, a car —a bit illegally— travels at a speed of 36 km/h. Another similar car overtakes it at a speed of 54 km/h. They are just next to each other when a child, who is 20 m ahead, runs to the road. Both drivers start to brake at the same moment, pushing the brakes at the same force. *a*) At what "remaining" speed does the faster car pass the child, if the other car just stops in front of the child? *b*) How does the result change if we consider that both drivers' reaction time is approximately 1 second? **G. 732.** News report (November 17, 2020): "The Crew Dragon spacecraft has arrived at the International Space Station (ISS). After a 27-hour totally autonomous flight it docked with the station, which was floating at a height of approximately 400 kilometres above the surface of the Earth." Estimate the following. *a*) How many times did the spaceship go around the Earth from its launch until it docked? *b*) What was the speed of the "floating" space station when the spaceship docked with it?

**P. 5283.** Three friends Sebi, Tóni and Zoli entered for the school's running competition held on Challenge Day. All of them covered the 2.4 km distance at a constant speed. When Tóni just covered 68% of the distance, Sebi had another three minutes to run. Zoli covered 20 cm more in each second than Sebi did, while he covered 10 cm less in each second than Tóni did. *a*) How much time elapsed between the moments when Zoli and Tóni reached the finish line? *b*) How far was Sebi from the finish line when Tóni reached it? **P. 5284.** The following can be read on the bottle of an alcoholic disinfectant solution: "Active ingredients: ethyl alcohol (70 V/V%)". The active ingredient contained by another type of solution is 67.9 m/m% ethanol (ethyl alcohol). Assuming that the amount of other additives is negligible, which solution has greater alcohol concentration? (The density of the ethanol-water mixture as a function of concentration can be found in tables.) Give a generally applicable relationship between the concentration of the solution, expressed

[*] `http://db.komal.hu/KomalHU/showpdf.phtml?tabla=FelHivatkoz&id=41643`

both in volume percent and in mass fraction, and density of the solution. **P. 5285.** A flat, disc-shaped object of mass $m$ is lying at rest on a rough horizontal surface. One end of a spring of force constant $k$ is attached to the centre of the disc, and then the other end is slowly pulled horizontally. Initially the spring is unstretched. The object stays at rest for a while and then it starts to move along a straight line. At the moment when the object starts to move the other end of the spring is fixed. *a*) What is the greatest speed of the object? *b*) How long does it take for the object to reach the maximum speed? *c*) How much distance does the object cover until it reaches its greatest speed? *d*) How will the object move afterwards, assuming that the spring remains straight? The coefficient of kinetic friction between the disc and the surface is $\mu$ and the coefficient of static friction is $\mu_0$ ($\mu_0 > \mu$). **P. 5286.** A uniform-density, thin, incomplete cylindrical shell of radius $R$ is placed onto a horizontal tabletop as shown in the *figure.* The angle at the "missing part" of the cylinder is $\varphi$. The cylindrical shell is displaced from its equilibrium position a bit and then released. Determine the period of the oscillation of the shell. Assume that friction is big enough and the shell does not slide during its oscillatory motion. *Data:* $R = 0.2$ m; $\varphi = \pi/3$. **P. 5287.** We have three resistors of resistance values 1 ohm, 2 ohms and 3 ohms, each rated at 1 watt. The three resistors are connected in all possible ways, such that some current flows through all of them in each connection. *a*) Between what values does the maximum allowed total power of the circuits vary? *b*) In which connection will the maximum allowed total dissipated power be exactly 2 watts? **P. 5288.** The walls of an aquarium are made of $d = 12$ mm thick glass of refractive index $n_\mathrm{g} = 3/2$. There is a fish swimming in the water. The refractive index of water is $n_\mathrm{w} = 4/3$. When the fish is observed from outside in the direction which is perpendicular to the wall of the aquarium, at what distance does that point of the fish seem to be which in reality is at a distance of exactly 20 cm from the external surface of the wall? **P. 5289.** A parallel monochromatic beam of light is incident on a high-resolution transmission grating which has vertical slits. In our experiment the light beam is perpendicular to the diffraction grating and the first-order images both towards the right and the left are diffracted by an angle of 30°. Then the diffraction grating is rotated by an angle of 30° about the line which coincides with the slit at the middle of the grating. At what directions will the diffracted beam exit the grating? **P. 5290.** A point-like negatively charged particle is projected from a point $P$ of a uniform electric field perpendicularly to the electric field at a velocity of $\boldsymbol{v}_0$. Uniform magnetic field is also present, which is perpendicular to both the electric field vector $\boldsymbol{E}$ and to the velocity $\boldsymbol{v}_0$. The two types of fields are separated by a plane, which is perpendicular to the electric field vector, as shown in the *figure.* What is the magnitude of the magnetic induction, if the particle returns back to point $P$? (The whole arrangement is in vacuum, and the effect of the gravitational force on the particle is negligible.) **P. 5291.** A carbon monoxide detector gives an alarm signal when the density of CO in the air reaches the value of $4 \cdot 10^{-6}$ kg/m$^3$. *a*) How many CO molecules does a person inhale in a single 500 cm$^3$ breath? *b*) What is the average energy of a single CO molecule in the lung at a temperature of 37 °C? *c*) What is the speed of an average-energy CO molecule? **P. 5292.** $^{14}$C isotope undergoes $\beta^-$ decay and its half life is 5568 years. In a certain amount of carbon, the initial activity of the isotope carbon-14 was 12 MBq. *a*) How many nuclides decayed in the first minute? *b*) How many nuclides decayed in the first ten thousand years? *c*) What was the initial mass of the carbon-14 isotope in the sample? *d*) How much time elapses until the mass of the carbon-14 isotope in the sample decreases to 1 $\mu$g? **P. 5293.** There are a lot of terminals at the top of a black box. It is known that inside the box, between any two pairs of terminals a resistor of unknown resistance is soldered. How can we measure the resistance of the resistor between two arbitrary chosen terminals if we have an ohm-meter and a large number of wires?