P. 5281. Legfeljebb mekkora töltésre tesz szert az a szigetelőállványra rögzített, 50 mm sugarú, kezdetben töltetlen fémgömbhéj, ha hosszú ideig olyan UV lámpával világítjuk meg, melynek legalacsonyabb kisugárzott hullámhossza 280 nm? A gömbhéj anyagának kilépési munkája 3,7 eV, a levegő vezetőképességétől eltekinthetünk.

(*4 pont*) Közli: *Vigh Máté,* Biatorbágy

P. 5282. Légpárnás asztalon mágneskorong mozog egy fémlap felett. Az örvényáramok hatására a sebességgel arányos fékezőerő hat a korongra. Egy alumíniumlap felett haladva a korong 30 cm út megtétele után áll meg, egy rézlap felett ugyanez a távolság csak 20 cm. Mekkora út megtétele után áll meg a mágneskorong, ha először egy 15 cm széles rézlap felett halad el, majd egy alumíniumlap felett folytatja mozgását? (A korong kezdősebessége mindhárom esetben ugyanakkora.)

(*6 pont*) Közli: *Gnädig Péter,* Vácduka

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Problems in Mathematics

New exercises for practice – competition K (see page 537): **K. 674.** In the backyard of aunt Ann, there are 120 animals: brown hens, white ducks, brown pigs and white rabbits. The number of white animals is 64, and the number of two-legged animals is 84. There are twice as many brown hens as white rabbits. How many of each species of animal live in aunt Ann's backyard? **K. 675.** A large company was giving a Christmas party to its employees. Some of them brought their spouses along. There were five times as many men present at the party as women. At 10 p.m., some husbands left for home with their wives, and thus the number of women dropped to one seventh of the number of men remaining. What fraction of the men left at 10 p.m.? **K. 676.** A 6×6 chessboard is tiled with eighteen 1×2 dominoes without overlaps. Show that it is possible to cut the chessboard with a straight line that will not cut across any domino stone. **K. 677.** The five elements of a number set S are pairwise added to produce the sums 0, 6, 11, 12, 17, 20, 23, 26, 32 and 37. Find the elements of S. (*Texas Mathematical Olympiad*) **K. 678.** There are 2020 coins lying on the table, lined up and showing heads, tails, heads, tails, ... alternating. In one move, it is allowed to reverse any three consecutive coins. With an appropriate sequence of such moves, is it possible to achieve that every coin should show tails?

New exercises for practice – competition C (see page 538): **Exercises up to grade 10: C. 1637.** In Dragonland, every seven-headed dragon blows fire, but not all seven-headed, fire-blowing creatures are dragons. According to the latest census figures, the number of dragons in Dragonland is equal to the number of fire-blowing creatures.

Is it true that every dragon has seven heads? **C. 1638.** Determine those non-regular triangles for which the orthocentre, the circumcentre, the incentre and two vertices are concyclic. **Exercises for everyone: C. 1639.** We have five numbers in mind. By selecting three numbers in every possible way and adding them together, we got the following sums: 41, 42, 44, 51, 52, 53, 54, 54, 55, 64. What are the five numbers? (Proposed by *S. Kiss,* Nyíregyháza) **C. 1640.** In a quadrilateral $ABCD$, let S denote the centroid of triangle ABC, and let P denote the centroid of triangle ACD. Prove that the line segment connecting the midpoints of diagonals AC and BD bisects the line segment SP. **C. 1641.** In the expansion of the power $(a+b+c)^{10}$, determine the coefficient of the term in $a^3b^2c^5$. (Proposed by *S. Kiss,* Nyíregyháza) **Exercises upwards of grade 11: C. 1642.** The opposite sides of a convex hexagon $ABCDEF$ are parallel, the distances separating the parallel pairs of sides are equal, and the angles at vertices A and D are right angles. Prove that the diagonals BE and CF enclose an angle of 45°. **C. 1643.** Without using a calculator, evaluate the expression $(\log_{10} 11)\cdot(\log_{11} 12)\cdot(\log_{12} 13)\cdot\ldots\cdot(\log_{99} 100)$.

New exercises – competition B (see page 539): **B. 5134.** Find all integers n for which the number $\sqrt{\frac{3n-5}{n+1}}$ is also an integer. (*3 points*) (Proposed by *M. Szalai,* Szeged) **B. 5135.** The feet of the altitudes drawn from vertices A, B, C of an acute-angled triangle ABC are A_1, B_1 and C_1, respectively; and the midpoints of the altitudes AA_1 and BB_1 are G and H, respectively. Prove that the circumscribed circle of triangle C_1GH passes through the midpoint F of side AB. (*4 points*) (Proposed by *B. Bíró,* Eger) **B. 5136.** The population of an island consists of underdogs and overlords. When a stranger visited the island, he was invited for dinner with a company of inhabitants. At the end, he asked each member of the company how many overlords were present. The underdogs all gave figures less than the true value and the overlords all gave figures larger than the true value. Is it true that the number of overlords can always be determined from the answers? (*5 points*) (Based on a problem of the *Dürer Competition*) **B. 5137.** Solve the following simultaneous equations over the set of real numbers: $x+y^2=z^3$, $x^2+y^3=z^4$, $x^3+y^4=z^5$. (*4 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5138.** Triangle ABC is not isosceles. The interior angle bisectors drawn from vertices A and B intersect the opposite sides at points A' and B', respectively. Prove that the perpendicular bisector of $A'B'$ will pass through the centre of the inscribed circle if and only if $AB'+BA'=AB$. (*5 points*) (Proposed by *L. Surányi,* Budapest) **B. 5139.** The diagonals of a convex quadrilateral $ABCD$ intersect at M. The area of triangle ADM is greater than that of triangle BCM. The midpoint of side BC of the quadrilateral is P, and the midpoint of side AD is Q, $AP+AQ=\sqrt{2}$. Prove that the area of quadrilateral $ABCD$ is smaller than 1. (*5 points*) **B. 5140.** There are 10 countries on an island, some of which share a border, and some do not. Each country uses a currency of its own. Every country operates a single exchange office, by the following rules: if you pay 10 units of the currency of that country, you will get 1 unit of each of the currencies of the bordering countries. Initially Arthur and Theodore each own 100 units of the currency of every country. Then each of them shops around the exchange offices of various countries in any order they like, and keeps exchanging money while they can (that is, while they have at least 10 units of a kind). Prove that Arthur and Theodore will have the same number of Bergengocian dollars at the end (the Bergengocian dollar is the currency of one of the countries on the island). (*6 points*) (Based on the idea of *G. Mészáros,* Budapest) **B. 5141.** Prove that $\sum_{i=0}^{n}\sum_{j=i}^{n}\binom{n}{i}\binom{n+1}{j+1}=2^{2n}$. (*6 points*) (Proposed by *Dávid Nagy,* Cambridge)

New problems – competition A (see page 541): **A. 789.** Let $p(x)=a_{21}x^{21}+a_{20}x^{20}+\cdots+a_1x+1$ be a polynomial with integer coefficients and real roots such that the absolute value of all of its roots are less than $1/3$, and all the coefficients of $p(x)$ are lying in the interval $[-2019a, 2019a]$ for some positive integer a. Prove that if this polynomial is reducible in $\mathbb{Z}[x]$, then the coefficients of one its factors are less than a. (Submitted by *Navid Safaei,* Tehran, Iran) **A. 790.** Andrew and Barry plays the following game: there are two heaps with a and b pebbles, respectively. In the first round Barry chooses a positive integer k, and Andrew takes away k pebbles from one of the two heaps (if k is bigger than the number of pebbles in the heap, he takes away the complete heap). In the second round the roles are reversed: Andrew chooses a positive integer and Barry takes away the pebbles from one the two heaps. This goes on, in each round the two players are reversing the roles. The player that takes the last pebble loses the game. Which player has a winning strategy? (Submitted by *András Imolay,* Budapest)

Problems in Physics

(see page 569)

M. 400. Investigate the position of the centre of mass of a pine branch, which was cut off near the trunk. That is, at what fraction of the length of the branch is the centre of mass? Carry out the measurement with the cut-off side branches as well. Take care, not to bend the branches too much. Compare the results. The pine branch can be a lowest branch of the Christmas tree, which had been cut off before the tree was put into its stand.

G. 725. The winding road in Bükk Mountains, which connects the cities Eger and Miskolc is approximately 50 km long. On a summer Sunday morning the traffic was heavy in both directions. Cars in both directions travelled at an average speed of 35 km/h, the oncoming cars passed each other on an average of one minute. Estimate the number of cars on the road at the same time (travelling in both directions). **G. 726.** The four inner cogwheels shown in the *figure* are moving round, whilst the outer one is at rest. (The motion of the cogwheels can be seen on the homepage.) *a*) Compare the periods of the cogwheels. *b*) Order the speeds of the centres of the cogwheels increasingly. **G. 727.** The length of a train is 93.5 m. The train starts from rest and travels at a constant acceleration along a straight railway. At the starting moment of the train a car, moving along a straight road parallel to the railway at a constant speed, is next to the end of the train, and after 14 seconds the car reaches the front of the train. After another 16 s, the car is again at the end of the train. *a*) What is the speed of the car? *b*) What is the acceleration of the train? *c*) How much distance does the car travel until the train finally passes it? **G. 728.** Some liquids – like raw milk or salad dressing made from olive oil and balsamic vinegar – when left to rest separate to their constituents. The oil will be on the top of the dressing and greasy cream will be on the top of the milk, while the total volume of the liquid does not change. How does the hydrostatic pressure at the bottom of the bottle into which the liquid is poured change when it is left to rest (increases, decreases or does not change) if the bottle *a*) is tapering upward; *b*) has a cylinder shape; *c*) is broadening out upward?

P. 5272. The four inner cogwheels shown in the *figure* are moving round, whilst the outer one is at rest. (The motion of the cogwheels can be seen on the homepage.) What are the values of the number of turns of the cogwheels labelled with the letters A, B and C if the smallest cogwheel labelled with D completes a full revolution in each second? **P. 5273.** A cuboid of mass $m = 0.5$ kg, base-side $a = 20$ cm and of height $b = 10$ cm is initially held at rest on the top of a right-angled inclined plane of mass M shown in the *figure.* The angle of elevation of the inclined plane is $\alpha = 30^\circ$ and its height is $h = 60$ cm. The cuboid is released at a certain moment. Friction is negligible everywhere. *a*) What is the ratio of the speeds of the two objects when the cuboid touches the ground? *b*) How