the plane. We know that for any k points ($k \geqslant 2$), it is possible to select two of them with distance at most 1. Show that the points can be covered with $k-1$ disks of unit radius. (*5 points*) **B. 5131.** Let H be an equilateral triangle of unit area, let O be a fixed point, and for any point P let H_P denote the triangle obtained from triangle H by a parallel shift with vector $\overrightarrow{OP}$. Consider the set N of points P for which the area of the intersection $H \cap H_P$ is at least $4/9$. What is the area of N? (*5 points*) (Based on the idea of *V. Vígh,* Székkutas) **B. 5132.** How many different strings of 2021 letters can be made of letters A, B and C such that the number of A's is even and the number of B's is of the form $3k+2$? (*6 points*) **B. 5133.** Given six points in the space, no four of which are coplanar, prove that they can be divided into two sets of three such that the two triangular plates spanned by the two sets of three points should intersect each other. (*6 points*)

New problems – competition A (see page 480): **A. 786.** In a convex set S that contains the origin it is possible to draw n disjoint unit circles such that viewing from the origin non of the unit circles blocks out a part of another (or a complete) unit circle. Prove that the area of S is at least $n^2/100$. (Submitted by: *Dömötör Pálvölgyi,* Budapest) **A. 787.** Let p_n denote the n^{th} prime number and define $a_n = \lfloor p_n \nu \rfloor$, where ν is a positive irrational number. Is it possible that there exist only finitely many k such that $\binom{2a_k}{a_k}$ is divisible by p_i^{10} for all $i = 1, 2, \ldots, 2020$? (Submitted by: *Abhishek Jha,* Delhi, India and *Ayan Nath,* Tezpur, India) **A. 788.** Solve the following system of equations: $x + \frac{1}{x^3} = 2y$, $y + \frac{1}{y^3} = 2z$, $z + \frac{1}{z^3} = 2w$, $w + \frac{1}{w^3} = 2x$.

Problems in Physics

(see page 506)

M. 399. Make different shapes of ice pieces in the freezer of a refrigerator and use them to measure the density of ice.

G. 721. In a building block set, every element is made of solid wood. Each of them has the same mass and has the shape of a cuboid. One side of each cuboid has a length of 6 cm, but the other two sides of the cuboids may be different. Luis put four blocks on top of one another, at the top there was a cube-shaped block. The whole bottom face of each block touched the face of the block below. Luis was amused by the tower, and also noticed that the tower is special for the pressure at the bottom face of each block is the same. Draw the sketch of the tower and also indicate in your figure the lengths of the sides of the cuboids. **G. 722.** In a pot, open at its top, water is boiled on a gas stove. Right after turning off the burner of the gas stove and after the flames ceased, white vapour cloud can be observed above the pot. Explain the phenomenon. **G. 723.** We have a converging and a diverging lens of powers of 5 dioptres and of -8 dioptres, respectively. A horizontal and parallel light beam enters into a dark room through a hole of a curtain, and a circular bright spot is created on the wall of the room. Which lens and where should be placed in order that the spot shrinks to a point? Then the other lens is put into the light beam as well. Where should it be placed in order that again a parallel beam of light be gained? Will this spot on the wall be smaller or greater than the original spot was? **G. 724.** In an experiment we can hear the humming sound of the iron core coil when 50 Hz AC current is given to it. What is the reason of the humming sound? What is the frequency of the humming sound?

P. 5261. The winner of stage 7 of the 2017 Tour de France was judged by photo finish. According to the photo finish Michael Kittel was only 6 millimetres ahead of Edvald Boason Hagen, the second, and their time difference was only 3 ten-thousands of a second.

a) At what speed did the cyclists travel at the finish? *b*) According to the official results the first three riders covered the 213.5 km distance during the same time of 5 hours 3 minutes and 18 seconds. What was the average speed of the riders for the whole distance? *c*) What may be the reason that the time of the first three riders was recorded to be the same? **P. 5262.** Formula One car drivers are participating in a race in which reaching the greatest speed is not the best tactic to win. A designated distance of $d = 1250$ m is to be covered at a constant speed, then each car has to stop at a deceleration of $a = 2$ m/s^2. The winner is the driver who can stop in the least time, measured from the start of the car. *a*) What should the speed of the winning car be at the constant speed stage of the motion, if the driver wants to stop in the least time? *b*) How much distance does the winning car cover in this case from the start to the stop? **P. 5263.** According to the Hungarian gun law only those guns can be possessed without a license whose muzzle energy (the kinetic energy of the bullet as it is expelled from the muzzle of the gun) does not exceed 7.5 J. The length of the barrel of our air gun, which just satisfies the above rule, is 480 mm, the diameter of the barrel is 4.5 mm, and the bullet fired is a spherical lead shot. *a*) What is the mean force which accelerates the bullet during a shot? What is the average pressure in the barrel? *b*) What is the muzzle speed of the bullet? *c*) What is the drag force exerted on the bullet short after it leaves the barrel? **P. 5264.** A racing car starts from rest and goes along a circular race track of radius 60 m. Its tangential acceleration is constant in the first four seconds of its motion, its magnitude is 6 m/s^2. *a*) Determine the angular speed at which the acceleration vector rotates with respect to the direction of the motion of the car. Sketch this angular speed as a function of the time. *b*) How much time elapses until this angular speed becomes the greatest? What is this greatest angular speed? **P. 5265.** A water polo player holds a ball above the water such that it just touches the surface of the water. The mass of the ball is 400 g, and its perimeter is 70 cm. At least how much work does the player have to do in order to push the ball totally under the water? **P. 5266.** A sample of ideal gas of degree of freedom f expands in an equilibrium process such that its pressure increases proportionally to the volume of the gas. By what factor will the absorbed heat by the gas be greater than the work done by the gas during the process? **P. 5267.** Steve is observing his eyeglasses. The lens of his glasses focuses the light of the Sun at a distance of 50 cm from the lens. He also observes that if the light of the Sun is reflected then two bright spots (foci) can be seen in front of the lens, one at a distance of 17 cm, and the other at a distance of 7 cm from the lens. What is the refractive index of the material of the lens? **P. 5268.** Two pieces of copper wires are soldered together, such that the two pieces form semicircles and together the wires form a circle of radius $r = 4$ cm. The diameters of the wires are $d_1 = 3$ mm and $d_2 = 1.5$ mm. To one of the solder points of the closed circle (A) and to the midpoint of the semicircle made of thinner wire (C) very long straight wires are connected (one to each point). Determine the magnetic induction at the centre of the circular wire, when the amperage in the straight wires is $I = 25$ A. **P. 5269.** What frequency sinusoidal AC supply is to be connected to the assembled elements shown in the *figure* in order that the arrangement have infinite resistance? **P. 5270.** The half life of the isotope radon-222 is 5508 minutes. How many days elapses until the activity of the radon sample decreases to one-tenth of its original value? **P. 5271.** A point-like object can move from point A to point B along the two paths shown in the *figure.* The distance between the two points is ℓ. In the case of *a*) the object moves along a horizontal straight path, and in the case of *b*) the object moves along a circular path in a vertical plane. The depth of the circular path is h. The initial speed of the object in both cases is v_0. Which motion lasts longer? (Air drag and friction is negligible.) *Data:* $v_0 = 1$ m/s, $\ell = 1$ m, $h = 2.5$ cm.