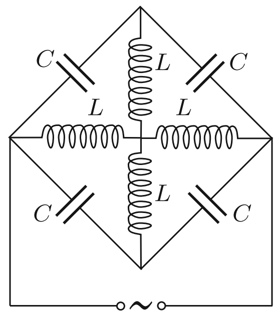
**P. 5269.** Mekkora frekvenciájú szinuszos váltóárammal szemben képvisel az *ábrán* látható összeállítás végtelen nagy ellenállást?

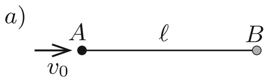
(*5 pont*) *Példatári feladat nyomán*

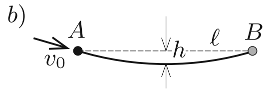



**P. 5270.** A radon 222-es izotópjának felezési ideje 5508 perc. Hány nap elteltével csökken egytizedére a radon aktivitása?

(*4 pont*) *Tankönyvi feladat nyomán*

**P. 5271.** Egy pontszerű test az *ábrán* látható kétféle útvonalon juthat el az $A$ pontból az $\ell$ távolságban lévő $B$ pontig. Az $a$) esetben a test vízszintes egyenes pályán mozog, a $b$) esetben pedig egy függőleges síkban elhelyezkedő, $h$ mélységű körív mentén. Mindkét mozgás kezdősebessége $v_0$. Melyik mozgás tart hosszabb ideig? (A súrlódás és a légellenállás elhanyagolható.)




*Adatok:* $v_0 = 1$ m/s, $\ell = 1$ m, $h = 2{,}5$ cm.

(*6 pont*) Közli: *Berke Martin,* Budapest

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**Elektronikus munkafüzet:** `https://www.komal.hu/munkafuzet`

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## Problems in Mathematics

**New exercises for practice – competition K** (see page 477): **K. 669.** Let us consider the set of 3-digit positive integers containing all the digits 1, 2, 3 exactly once. Find the smallest positive integer that contains each number from the previous set as consecutive digits. **K. 670.** Grandma bought two candles: the red candle was 2 cm longer than the blue one. On All Saints' Day she lit the red candle at 5:30 p.m. then she lit the blue candle at 7 p.m. and let them burn all the way down. The two candles were equal in length at 9:30 p.m. The blue candle burned out at 11 p.m and the red one finished at 11:30 p.m. What was the initial length of the red candle? **K. 671.** We know that the first five terms of an increasing arithmetic sequence are all positive primes. Find the smallest prime at the $5^{\text{th}}$ position. **K. 672.** A garden is divided into 16 patches as shown in the *figure.* In each patch, either roses or tulips or daisies or gerberas are grown: only one type of flower in each, but every row, every column, and both diagonals contain every type of flower. In how many different ways is it possible to arrange the flowers in this way? (Two arrangements are considered different if there exists a patch that contains a different kind of flower.) **K. 673.** The students in a class (we do not know how many of them there are) decided that everyone would buy some small present to everyone else for Christmas, and they would also buy some present together for each of their 11 teachers. Unfortunately, the Christmas party was cancelled. Then they decided to divide the presents equally among all the siblings of the students. (Each sibling gets the same present.) Was that possible if the total number of siblings was 15?

**New exercises for practice – competition C** (see page 478): **Exercises up to grade 10: C. 1630.** The numbers 1 to 32 are written in the white fields of a chessboard, using each number once. Then the sum of the numbers in the adjacent fields is entered in each black field. What are the smallest and largest possible values of the sum of the numbers in the black fields? **C. 1631.** Let $AB$ be a chord in a unit circle. Triangle $ABC$ is right-angled at $B$, and vertex $C$ lies on the circle. Triangle $ABD$ is isosceles right-angled, and $AB$ is the hypotenuse. How long is the chord $AB$ if the two triangles have equal areas? What is this area? **Exercises for everyone: C. 1632.** How many different infinite arithmetic sequences of positive integer terms are there in which 24, 744 and 2844 all occur? Two arithmetic sequences are considered different if they have different first terms or different common differences. **C. 1633.** Let $P$ be an interior point of one side of a unit square. Consider all parallelograms with one vertex at $P$, and one on each side of the square. Prove that if $P$ is not the midpoint of the side then ($i$) there are exactly two rectangles among these parallelograms, and ($ii$) the sum of the areas of these two rectangles is 1. **C. 1634.** Prove the following inequality: $\frac{1}{4}+\frac{1}{28}+\frac{1}{70}+\cdots+\frac{1}{(3k-2)(3k+1)}+\cdots+\frac{1}{2017\cdot 2020}<\frac{1}{3}$. **Exercises upwards of grade 11: C. 1635.** Given two intersecting circles, construct* a secant through one of the intersection points such that the segment bounded by the two circles is divided $1:2$ by the intersection point. Write down and explain the steps of the construction. (Elementary steps like bisecting an angle or reflecting a point in a line do not need to be described in detail.) **C. 1636.** The Hungarian poet Dezső Kosztolányi spent a few weeks in Paris when he was a student. When he was given for change a ten-centime coin not in circulation any more, he wanted to give it away. He did not succeed, which he explained to himself by the expression on his face revealing his intentions. Therefore he decided to get 9 valid ten-centime coins, mix them with the worthless coin in his pocket, and by not looking at them he pays with one of them in a shop. He continued doing so until he had a single coin in his pocket: the coin out of circulation. What is the probability of this?

**New exercises – competition B** (see page 479): **B. 5126.** Prove that if $n\geqslant 3$, then there exist $n$ distinct positive integers such that the sum of their reciprocals is 1. (*3 points*) **B. 5127.** Given a convex angle and a line segment of length $k$, determine the locus of those points inside the angle through which there exists a line cutting off a triangle of perimeter $k$ from the angle. (*4 points*) **B. 5128.** Find all pairs of relatively prime integers $(x,y)$ such that $x^2+x=y^3+y^2$. (*4 points*) (Proposed by *L. Surányi,* Budapest) **B. 5129.** Two players are taking turns in selecting one of the coefficients $a$, $b$ and $c$ of the polynomial $x^3+ax^2+bx+c$, and giving it some integer value of their choice. Prove that the starting player can achieve that (after the three steps) all three roots of the polynomial should be integers (i.e. that the polynomial can be expressed as a product of three polynomials of integer coefficients). (*3 points*) **B. 5130.** There are $n$ points in

*with straight edge and compasses on paper, or with appropriate geometric construction software the plane. We know that for any $k$ points ($k \geqslant 2$), it is possible to select two of them with distance at most 1. Show that the points can be covered with $k-1$ disks of unit radius. (*5 points*) **B. 5131.** Let $H$ be an equilateral triangle of unit area, let $O$ be a fixed point, and for any point $P$ let $H_P$ denote the triangle obtained from triangle $H$ by a parallel shift with vector $\overrightarrow{OP}$. Consider the set $N$ of points $P$ for which the area of the intersection $H \cap H_P$ is at least $4/9$. What is the area of $N$? (*5 points*) (Based on the idea of *V. Vígh,* Székkutas) **B. 5132.** How many different strings of 2021 letters can be made of letters A, B and C such that the number of A's is even and the number of B's is of the form $3k+2$? (*6 points*) **B. 5133.** Given six points in the space, no four of which are coplanar, prove that they can be divided into two sets of three such that the two triangular plates spanned by the two sets of three points should intersect each other. (*6 points*)

**New problems – competition A** (see page 480): **A. 786.** In a convex set $S$ that contains the origin it is possible to draw $n$ disjoint unit circles such that viewing from the origin non of the unit circles blocks out a part of another (or a complete) unit circle. Prove that the area of $S$ is at least $n^2/100$. (Submitted by: *Dömötör Pálvölgyi,* Budapest) **A. 787.** Let $p_n$ denote the $n^{\text{th}}$ prime number and define $a_n = \lfloor p_n \nu \rfloor$, where $\nu$ is a positive irrational number. Is it possible that there exist only finitely many $k$ such that $\binom{2a_k}{a_k}$ is divisible by $p_i^{10}$ for all $i = 1, 2, \ldots, 2020$? (Submitted by: *Abhishek Jha,* Delhi, India and *Ayan Nath,* Tezpur, India) **A. 788.** Solve the following system of equations: $x + \frac{1}{x^3} = 2y$, $y + \frac{1}{y^3} = 2z$, $z + \frac{1}{z^3} = 2w$, $w + \frac{1}{w^3} = 2x$.

## Problems in Physics

(see page 506)

**M. 399.** Make different shapes of ice pieces in the freezer of a refrigerator and use them to measure the density of ice.

**G. 721.** In a building block set, every element is made of solid wood. Each of them has the same mass and has the shape of a cuboid. One side of each cuboid has a length of 6 cm, but the other two sides of the cuboids may be different. Luis put four blocks on top of one another, at the top there was a cube-shaped block. The whole bottom face of each block touched the face of the block below. Luis was amused by the tower, and also noticed that the tower is special for the pressure at the bottom face of each block is the same. Draw the sketch of the tower and also indicate in your figure the lengths of the sides of the cuboids. **G. 722.** In a pot, open at its top, water is boiled on a gas stove. Right after turning off the burner of the gas stove and after the flames ceased, white vapour cloud can be observed above the pot. Explain the phenomenon. **G. 723.** We have a converging and a diverging lens of powers of 5 dioptres and of −8 dioptres, respectively. A horizontal and parallel light beam enters into a dark room through a hole of a curtain, and a circular bright spot is created on the wall of the room. Which lens and where should be placed in order that the spot shrinks to a point? Then the other lens is put into the light beam as well. Where should it be placed in order that again a parallel beam of light be gained? Will this spot on the wall be smaller or greater than the original spot was? **G. 724.** In an experiment we can hear the humming sound of the iron core coil when 50 Hz AC current is given to it. What is the reason of the humming sound? What is the frequency of the humming sound?

**P. 5261.** The winner of stage 7 of the 2017 Tour de France was judged by photo finish. According to the photo finish Michael Kittel was only 6 millimetres ahead of Edvald Boason Hagen, the second, and their time difference was only 3 ten-thousands of a second.