P. 5260. Vízszintes tengelyű, rögzített hengeren súrlódó fonalat vetünk át. Ha a fonál bal oldali végére m tömegű nehezéket, a jobb oldalira pedig $3m$ tömegűt akasztunk, akkor az álló helyzetből elengedett testek 2 m/s^2 nagyságú gyorsulással mozognak.

a) Mekkora gyorsulással mozognak a testek, ha mindkét oldalon először megduplázzuk, majd megháromszorozzuk a tömegüket?

b) Mekkora gyorsulással mozognak a testek, ha a jobb oldalon meghagyjuk a $3m$ nagyságú tömeget, de a bal oldali fonálvégre $8m$ tömegű testet akasztunk?

c) Hogyan válasszuk meg a bal oldali fonálvégre akasztott test tömegét, miközben a jobb oldalon megmarad a $3m$ tömeg, hogy elengedés után a rendszer nyugalomban maradjon?

A fonál nagyon könnyű, továbbá a fonál és a henger közötti csúszási súrlódás együtthatója megegyezik a tapadási súrlódás együtthatójával.

(*6 pont*) Közli: *Honyek Gyula,* Veresegyház

Beküldési határidő: 2020. november 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 70. No. 7. October 2020)

Problems in Mathematics

New exercises for practice – competition K (see page 416): **K. 664.** We have six coins, four of which weigh 100 grams each, and the remaining two weigh 99 grams each. With the help of an equal-arm balance and no weights, what is the minimum number of measurements that are sufficient to identify one of the lighter coins? **K. 665.** Some toy robots are lining up on one side of a street. In one move, we can instruct exactly three robots to cross the street. For what number of robots can we make all the robots line up on the opposite side? **K. 666.** How many six-digit multiples of 182 are there in which the three-digit number formed by the first three digits is equal to the three-digit number formed by the last three digits? **K. 667.** Start with a positive integer. In each move, take the half of the number if it is even, or add 1 to the number if it is odd. The sequence of moves terminates if it reaches the number 1. *a*) Is it true that whatever the starting number is, it is always possible to reach 1 sooner or later (with a finite number of moves)? *b*) Is it true that at most 30 moves are sufficient to reach 1 if we start from a four-digit number? **K. 668.** *a*) How many isosceles triangles are there for which the length of the legs is 13 cm and the area is 60 cm^2? *b*) How many right-angled triangles are there for which the legs are even integers, and the area is 60 cm^2?

New exercises for practice – competition C (see page 417): **Exercises up to grade 10: C. 1623.** Let m be a positive integer. Show that *a*) there exist three m-digit powers of 2; *b*) there exist at most four m-digit powers of 2. (*Brazilian problem*) **C. 1624.** Point P of side AB in a square $ABCD$ is connected to D, and point Q of side BC is connected to A. The intersection of the resulting line segments is denoted by R. The area of triangle ARD is 1200, the area of triangle APR is 600, and the area of quadrilateral $PBQR$ is $3380-240\sqrt{95}$ units of area. What is the area of quadrilateral $RQCD$? (Proposed by *L. Németh,* Fonyód) **Exercises for everyone: C. 1625.** Prove that every selection of five one-digit positive integers contains a few numbers whose sum is divisible by 10. **C. 1626.** Let F denote the midpoint of side BC in an acute-angled triangle ABC, and let T be the foot of the altitude drawn from B. Prove that if $\angle FAC=30^\circ$ then $AF=BT$. (Based on the idea of *S. Róka,* Nyíregyháza) **C. 1627.** Prove that if a, b, c are real numbers, such that $a+b+c>0$, $ab+bc+ca>0$ and $abc>0$, then $a>0$, $b>0$ and $c>0$. (Proposed by *S. Róka,* Nyíregyháza) **Exercises upwards of grade 11: C. 1628.** Find two distinct positive integers n for which $4^n+4^9+4^{100}$ is a perfect square. **C. 1629.** A sphere passes through four vertices of one face of a cube, and is tangent to the opposite face. Determine the radius of the sphere if the edge of the cube is 8 units long. (*Croatian problem*)

New exercises – competition B (see page 418): **B. 5118.** Is it possible that x, $\frac{14x+5}{9}$ and $\frac{17x-5}{12}$ are all integers? (*3 points*) **B. 5119.** In an acute-angled triangle ABC, a tangent is drawn to the inscribed circle, parallel to side BC. The tangent intersects side AC at point D. F is the orthogonal projection of point D onto the side BC. Show that $AB=AD+BF$. (*3 points*) **B. 5120.** The positive integers are coloured in the following manner: the colour of $a+b$ is always uniquely determined by the colours of a and b; that is, if the colour of a and a' is the same, and the colour of b and b' is the same, then $a+b$ and $a'+b'$ also have the same colour. Prove that if there is a colour that is used more than once then the colouring becomes periodic from some number onwards. (*4 points*) **B. 5121.** Solve the following simultaneous equations, where $x_1,x_2,\ldots,x_n$ are positive real numbers, and n is a positive integer: $x_1+x_2+\dots+x_n=9$, $\frac{1}{x_1}+\frac{1}{x_2}+\dots+\frac{1}{x_n}=1$. (*4 points*) **B. 5122.** ErWin Layup is the best penalty taker of all times in the basketball league of Nowhereland. Although he missed the very first penalty throw of his career, altogether he has only missed 2020 out of his total of 222 222 throws. Statisticians in Nowhereland consider a basketball penalty throw *interesting* if the ratio of successful penalty throws to all penalty throws, calculated immediately after the throw and expressed as a percentage, is a positive integer. (For example, if a player scores 12 out of a total of 40 throws then his last throw is interesting, since $\frac{12}{40}\cdot 100=30\in\mathbb{N}^+$, while the following throw, which is the 41st, cannot be interesting, whether successful or not.) What is the minimum number of interesting penalty throws that ErWin Layup may have had? (*5 points*) **B. 5123.** Ann and Barbara divided between themselves the 81 cards of the game of SET*; Ann received 40 cards and Barbara received 41. Each girl counted the number of ways they can form a SET of three cards out of the cards held by her. What may be the sum of the numbers they obtained? (*6 points*) **B. 5124.** The base of a right pyramid is a square $ABCD$, and the apex of the pyramid is E. The skew edges AB and CE are connected by a transversal that is normal to both of them. The feet of the normal transversal are point P on the line segment AB, and point Q on the line segment CE. Given that Q bisects the edge CE, determine the ratio $AP:PB$, and calculate the angle enclosed between the lateral faces and the base of the pyramid. (*5 points*) **B. 5125.** The centre of the circumscribed circle of a cyclic quadrilateral $ABCD$ is O. The rays AB and DC intersect at point E. In the circle BCE, the point diametrically opposite to E is F. Show that the lines AC, BD and OF are concurrent. (*6 points*)

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*`https://www.setgame.com/sites/default/files/instructions/SET%20INSTRUCTIONS%20-%20ENGLISH.pdf`.

New problems – competition A (see page 419): **A. 783.** A *polyomino* is a figure which consists of unit squares joined together by their sides. (A polyomino may contain holes.) Let $n \geqslant 3$ be a positive integer. Consider a grid of unit square cells which extends to infinity in all directions. Find, in terms of n, the greatest positive integer C which satisfies the following condition: For every colouring of the cells of the grid in n colours, there is some polyomino within the grid which contains at most $n-1$ colours and whose area is at least C. (Submitted by *Nikolai Beluhov,* Stara Zagora, Bulgaria and *Stefan Gerdjikov,* Sofia, Bulgaria) **A. 784.** Let n, s, t be positive integers and $0<\lambda<1$. A simple graph on n vertices with at least λn^2 edges is given. We say that $(x_1,\dots,x_s,y_1,\dots y_t)$ is a*good insertion,* if letters x_i and y_j denote not necessarily distinct vertices and every x_iy_j is an edge of the graph ($1 \leqslant i \leqslant s$, $1 \leqslant j \leqslant t$). Prove that the number of good insertions is at least $\lambda^{st}n^{s+t}$. (Submitted by *Kada Williams,* Cambridge) **A. 785.** Let $k \geqslant t \geqslant 2$ positive integers. For integers $n \geqslant k$ let p_n be the probability that if we choose k from the first n positive integers randomly, any t of the k chosen integers have greatest common divisor 1. Let q_n be the probability that if we choose $k-t+1$ from the first n positive integers the product is not divisible by a perfect t-th power that is greater then 1. Prove that sequences p_n and q_n converge to the same value. (Submitted by *Dávid Matolcsi,* Budapest)

Problems in Physics

(see page 442)

M. 398. Measure the rolling resistance between a cylinder and the ground. Use two different cylinders of the same radius and carry out the measurement for two different surfaces. (The two different cylinders can be for example a paper cylinder of a roll of plastic wrap, and an aluminium foil roll, whilst the two different surfaces can be the floor of the room with and without a soft carpet.) Investigate how much the deceleration of the cylinder can be considered constant.

G. 717. A bat flies parallel to the wall of a cave at a speed of 45.0 m/s. It emits a short ultrasound signal, the echo of which is heard after 0.120 s. How far does the bat fly from the wall? The speed of ultrasound in the cave is 333 m/s. **G. 718.** Suppose the material of the Sun consists of carbon and oxygen. (In the old days, this idea came up seriously.) At most how much would the total lifespan of the Sun be if the coal burns perfectly and the energy radiated in a unit time is the same as it is now? (In the calculations, let us use the actual mass of the Sun.) **G. 719.** A closed beverage can of size 330 ml is floating in water. The can is made of aluminium, and the mass of the empty can is 13 g. How many millilitres of gas is in the closed can, if it contains exactly 330 ml of soft drink of density approximately the same as that of water? **G. 720.** In the Tour de France cycling race, the riders go uniformly at a speed of 50 km/h on a horizontal road. The distance between the peloton and the breakaway riders is 1 km. When the riders reach an approximately 5 km long climb their speed soon decreases to 40 km/h, and when they move downwards also along a distance of 5 km their speed soon increases to 60 km/h. Sketch the distance between the peloton and the breakaway group as a function of time from the moment when the breakaway reaches the climb, until the moment it reaches the end of the downhill slope.

P. 5250. A car travels at a constant speed along a long, straight road. Consider a point on the rim of the wheel of the car. Investigate the whether *a*) the average speed of this point is greater, smaller or equal to the speed of the car; *b*) the magnitude of the average velocity of this point is greater, smaller or equal to the speed of the car. **P. 5251.** A small body of mass m is released from rest at point A of a fixed prism shown in the *figure.* The body slides frictionlessly along the straight slope on the left side and along the