get? (*5 points*) **B. 5116.** Let $a, b, c > 0$ and $x, y, z \geqslant 0$. Prove that if $x + aby \leqslant a(y + z)$, $y + bcz \leqslant b(z + x)$, and $z + cax \leqslant c(x + y)$, then $x = y = z = 0$ or $a = b = c = 1$. (*6 points*) (Proposed by *G. Stoica,* Saint John, Kanada) **B. 5117.** The points A, B, C, D (in this order) lie on the same line. On the same side of the line, a regular triangle is drawn on each of the line segments AB, BC and CD, with the third vertices being E, F, and G, respectively. Let the distances between the adjacent points on the line be $AB = a$, $BC = b$, $CD = c$. Show that the measure of $\angle EFG$ equals 120° if and only if $a + c = b$ or $\frac{1}{a} + \frac{1}{c} = \frac{1}{b}$. (*6 points*)

New problems – competition A (see page 356): **A. 780.** We colored the n^2 unit squares of an $n \times n$ square lattice such that in each 2×2 part at least two of the four unit squares has the same color. What is the largest number of colors we could have used? (Based on a problem of the *Dürer Competition*) **A. 781.** We want to construct an isosceles triangle using a compass and a straightedge. We are given two of the following four data: the length of the base of the triangle (a), the length of the leg of the triangle (b), the radius of the inscribed circle (r) and the radius of the circumscribed circle (R). In which of the six possible cases will we definitely be able to construct the triangle? (Proposed by *György Rubóczky,* Budapest) **A. 782.** Prove that the edges of a simple planar graph can always be oriented such that the outdegree of all vertices is at most three. (*UK competition problem*)

Problems in Physics

(see page 378)

M. 397. By measuring the temperature of a smoked metal plate (which was smoked with the flames of a candle) determine the amount of energy absorbed by a unit area surface perpendicular to the radiation emitted by the Sun in a unit of time. (Take the specific heat capacity of the metal from a table.)

G. 713. An 80-kg physics teacher makes a first class lever from a 6-m long strong wooden plank of mass 40 kg, with which he demonstrates to his students that he can lift a load of mass even up to 500 kg. Where should he place the fulcrum of the lever if he wants to lift the load up to the highest possible position such that he gently puts his total weight onto the other end of the plank? The vertical line through the centre of mass of the load is at a distance of 20 cm from the end of the plank. **G. 714.** The Earth's ice caps and glaciers currently contain approximately 30 000 000 km^3 of ice. Let us estimate how much the sea level of the oceans and seas would rise if all this huge amount of ice melted. **G. 715.** A circuit consists of three resistors and a battery as shown in the *figure.* *a*) What is the current flowing through each resistor and the voltage across them? *b*) How do these values change if we connect a lot of ("infinitely many") 1 kΩ resistors in parallel to the two resistors, already connected in parallel? *c*) What will the currents through the original three resistors and voltages across them be, if we connect a lot of ("infinitely many") 1 kΩ resistors in series with the 5 kΩ resistor? **G. 716.** A grenade fired from a cannon explodes into two pieces of equal mass at the top of its trajectory, when its speed is 100 m/s. One piece starts to move vertically upwards at a speed of 50 m/s. In what direction and at what speed does the other piece start? (The mass of the explosive in the grenade is negligible.)

P. 5240. How many litres of air is displaced from a room of sides 6 m × 5 m × 3 m, if the temperature of the air increases from 27 °C to 30 °C, while the pressure decreases by 0.5%? **P. 5241.** On May 14, 1962, due to the strong South West wind the water level of lake Balaton near city Keszthely decreased by 45 cm in 9 hours, while near the village

Alsóörs the water level increased by 51 cm. Estimate the order of the power required of the wind to raise the level of water. (It is allowed to use relevant data found in the internet.) **P. 5242.** Spherical raindrops of diameter 2 mm are floating in a cloud. At what speed does the air of density 1 kg/m^3 flow upwards in the cloud? (Air drag is proportional to the square of the speed.) **P. 5243.** In a sports hall, handball players practice starting by running parallel to the wall of the room to catch a ball thrown against the wall. One of the players runs 3 meters from the wall at a constant speed of 5 m/s. At least at what speed with respect to the hall does he or she have to throw the ball in order to catch it at the height of the throw? Consider the collision of the ball with the wall to be perfectly elastic. **P. 5244.** The energy dissipated by a specific type of elementary particle while it is moving in some solid is proportional to the distance covered by the particle, and finally it stops somewhere. Particles with an initial speed of $v_0 = 10^7$ m/s penetrate into some lighter material to a depth of $s_1 = 3$ cm, whereas they penetrate into some more dense material to a depth of $s_2 = 2$ cm. At what depth can these particles, with the same initial speed, go into the lighter material after passing through a $d = 1.5$ cm thick layer of the more dense material? **P. 5245.** A cargo plane travels 11 km above the equator at a speed of 1000 km/h, first west and then east. The mass of a heavy object on board is measured on both occasions using a spring balance certified at the airport. The difference between the two measured values is 1 kg. What is the mass of the object in reality? **P. 5246.** At the bottom of lake Hévíz, where the spring breaks out from the rock, a spherical bubble is generated from the mud. The bubble is supposed to keep its spherical shape and its diameter increases by 50% while it moves up to the surface of the water. The temperature of the water is the same everywhere. What is the depth of the water above the mud layer? **P. 5247.** On each of the opposite faces of a rectangular aquarium there is a circular hole covered by a thin spherical cap-shaped piece of glass, as shown in the *figure.* The common principal axis of the caps is horizontal. The radius of curvature of the concave cap – the one depressed into the aquarium – is r, and that of the convex cap – bulging outward from the aquarium – is $2r$. The topmost points of both caps are below the surface of the water in the aquarium. The refractive index of water is $n = 4/3$. (The angle between the principal axis of the spherical glass cap, and the radius drawn from the centre of the sphere to a point on the perimeter of the circular base of the cap is small.) *a*) What is the distance between the two faces of the aquarium, containing the glass caps, if a parallel beam of light entering horizontally to one of the spherical caps emerges from the other glass cap as a parallel and horizontal beam of light? *b*) What is the ratio of the diameters of the two spherical caps d_2 to d_1, if a horizontal light beam entering the aquarium through any of the spherical caps exits entirely through the other spherical glass cap? *c*) There is a tiny fish at the common principal axis in the middle of the aquarium. Where can this fish be observed, when viewed through one of the glass caps on one side, then through the other? **P. 5248.** A wire of length 4ℓ was bent at right angles at its two quadrisecting points, next to the ends of the wire. Where should another piece of 2ℓ-long wire of the same type be connected to the first wire, if the equivalent resistance of the wire system is to be the same as the resistance of the wire of length 2ℓ? **P. 5249.** The length of an AA rechargeable battery is 5 cm, and its diameter is 1.4 cm. *a*) How much energy is stored in the battery rated at 1.2 V and 2800 mAh? *b*) To what speed could such a battery speed up, if all of its stored energy was converted to its kinetic energy? The mass of the battery is 17 g. *c*) By what factor would the energy needed to heat the same volume of water from 20 °C to 100 °C be less than the above calculated energy? *d*) How much energy is there in the same volume of granulated sugar of density of approximately 0.77 g/cm^3? The energy content of sugar is 1680 kJ/100 grams.