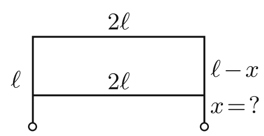
c) Az optikai tengelyen, az akvárium közepén van egy piciny halacska. Hol látható ez az egyik, illetve a másik oldali gömbsüvegen át nézve?

(*5 pont*) Közli: *Zsigri Ferenc,* Budapest

P. 5248. Egy 4ℓ hosszúságú ellenálláshuzalt a két negyedelőpontjában derékszögben meghajlítottunk. Hol kell ehhez hozzákötni a 2ℓ hosszúságú, ugyanebből a huzalból levágott vezetőt, ha azt akarjuk, hogy a huzalvégek között kialakuló eredő ellenállás megegyezzen egyetlen 2ℓ hosszúságú vezető ellenállásával?



(*4 pont*) *Példatári feladat nyomán*

P. 5249. Az AA jelű akkumulátor hossza 5 cm, átmérője 1,4 cm.

a) Mekkora energiát tárol egy 1,2 V-os, 2800 mAh-s akku?

b) Mekkora sebességre gyorsulna fel ez a 17 grammos akku, ha az eltárolt energiáját teljesen a saját mozgási energiájává alakítaná?

c) Hányszor kevesebb energiával lehetne ugyanekkora térfogatú vizet 20 °C-ról 100 °C-ra melegíteni?

d) Mennyi energia van ugyanekkora térfogatú kristálycukorban, amelynek sűrűsége kb. 0,77 g/cm^3, energiatartalma pedig 1680 kJ/100 gramm?

(*4 pont*) Közli: *Vass Miklós,* Budapest

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Problems in Mathematics

New exercises for practice – competition K (see page 353): **K. 659.** How many different quadrilaterals are there whose vertices are selected from the vertices of a regular nonagon so that the quadrilateral contains the centre of the nonagon in its interior? (Congruent quadrilaterals are not considered different.) **K. 660.** The squares in the *figure* were filled in with the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and then the sum of the numbers in the two adjacent squares was entered in each circle. Finally, the numbers in a few circles were deleted, and the squares were shaded. *a*) Which numbers are missing from the blank circles? *b*) Enter the original number in each square. **K. 661.** The sides of a regular octagon $ABCDEFGH$ are 2 units long. Two squares, $BCIM$ and $GHNP$ are constructed on sides BC and GH, inside the octagon. Show that the points N and M coincide. **K. 662.** The first four terms of a sequence are all 1. From the fifth term onwards, each term is obtained by adding the two terms that are four positions and three positions back from that term. How many even numbers are there among the first 150 terms of the sequence? **K. 663.** The sum of the squares of three consecutive integers equals the sum of the squares of the following two integers. What may be these five consecutive numbers?

New exercises for practice – competition C (see page 354): **Exercises up to grade 10: C. 1616.** Solve the equation $x+\dfrac{1}{y+\dfrac{1}{\frac{5z}{4}+\frac{13}{6v}}}=\dfrac{135}{113}$ where x, y, z, v are positive integers. (Proposed by *B. Bíró,* Eger) **C. 1617.** Is it possible to arrange four 2×8 rectangles in the interior of a 7×15 rectangle as shown in the *figure,* without overlapping? (The figure is not to scale.) **Exercises for everyone: C. 1618.** Prove that $\frac{2}{3}\leqslant a_{n+1}-a_n<1$ holds for all the terms of the sequence $a_n=\frac{(n-1)n}{n+1}$, with $n\geqslant 1$. **C. 1619.** From the midpoint of each side of an acute-angled triangle, drop perpendiculars to the other two sides. Prove that the area of the hexagon formed by the perpendiculars is half the area of the triangle. (*Croatian problem*) **C. 1620.** Jack Russell fashioned a springboard in his back yard. He carried out measurements to determine that by bending the springboard x decimetres below horizontal, he could jump to a height of $0.5x^2+ax+b$ decimetres. Unfortunately, he forgot the values of a and b. He only remembers that bending by 10 cm let him jump 35 cm, and a deformation four times as large resulted in a jump four times as high. What may have been Jack's measured values of a and b? **Exercises upwards of grade 11: C. 1621.** The lengths of the sides of a trapezoid circumscribed about a circle are integers that are consecutive terms of an arithmetic progression in some order. Given that the radius of the incircle and the length of the shorter base are both 6, how long are the other three sides? (Proposed by *L. Németh,* Fonyód) **C. 1622.** Prove that the area bounded by the graphs of the functions $y=1-|x-1|$ and $y=|2x-a|$ is less than $\frac{1}{3}$ for $1<a<2$. (*Croatian problem*)

New exercises – competition B (see page 355): **B. 5110.** The tangents drawn to the inscribed circle of an isosceles triangle, parallel to the sides, cut off three small triangles. Prove that in the small triangles lying on the base of the large triangle, the heights drawn to the base are equal to the radius of the inscribed circle of the large triangle. (*3 points*) **B. 5111.** Let a and b be real numbers such that $a+b=1$ and $a^2+b^2=2$. Find the value of a^8+b^8. (*3 points*) (Based on the idea of *M. Szalai,* Szeged) **B. 5112.** A deck of card consists of p red cards and k blue cards. In how many different ways is it possible to select some of the cards so that the number of red cards should be n more than the number of blue cards? (*4 points*) **B. 5113.** Let a, b and c denote some given, pairwise relatively prime positive integers. Prove that the equation $x^a+y^b=z^c$ has infinitely many solutions (x,y,z) where x, y and z are positive integers. (*5 points*) **B. 5114.** A unit cube $ABCDEFGH$ (see the *figure*) is cut by a plane $\mathcal{S}$ that intersects the edges AB and AD at the points P and Q respectively, such that $AP=AQ=x$ $(0<x<1)$. Let the common point of the edge BF and $\mathcal{S}$ be R. What is the distance BR if $\angle QPR=120^\circ$? (*4 points*) **B. 5115.** Ali has n coins in his purse, and Baba has $n-1$ purses, initially all empty. Baba is playing the following game: he divides the coins (all in the same purse at start) into two purses, with a_1 coins in one of them and b_1 coins in the other $(a_1,b_1>0)$, and then he writes the product a_1b_1 on a blackboard. Then he continues in the same way: in the kth move $(k=2,3,\ldots)$ he selects a purse containing at least two coins, divides them between two empty purses, with a_k coins in one of them and b_k in the other $(a_k,b_k>0)$, and writes the product a_kb_k on the board. The game terminates when there is 1 coin in each purse. Then Ali gives as many coins to Baba as the sum of all the products a_kb_k on the board. What is the maximum number of coins that Baba may get? (*5 points*) **B. 5116.** Let $a, b, c > 0$ and $x, y, z \geqslant 0$. Prove that if $x + aby \leqslant a(y + z)$, $y + bcz \leqslant b(z + x)$, and $z + cax \leqslant c(x + y)$, then $x = y = z = 0$ or $a = b = c = 1$. (*6 points*) (Proposed by *G. Stoica,* Saint John, Kanada) **B. 5117.** The points A, B, C, D (in this order) lie on the same line. On the same side of the line, a regular triangle is drawn on each of the line segments AB, BC and CD, with the third vertices being E, F, and G, respectively. Let the distances between the adjacent points on the line be $AB = a$, $BC = b$, $CD = c$. Show that the measure of $\angle EFG$ equals 120° if and only if $a + c = b$ or $\frac{1}{a} + \frac{1}{c} = \frac{1}{b}$. (*6 points*)

New problems – competition A (see page 356): **A. 780.** We colored the n^2 unit squares of an $n \times n$ square lattice such that in each 2×2 part at least two of the four unit squares has the same color. What is the largest number of colors we could have used? (Based on a problem of the *Dürer Competition*) **A. 781.** We want to construct an isosceles triangle using a compass and a straightedge. We are given two of the following four data: the length of the base of the triangle (a), the length of the leg of the triangle (b), the radius of the inscribed circle (r) and the radius of the circumscribed circle (R). In which of the six possible cases will we definitely be able to construct the triangle? (Proposed by *György Rubóczky,* Budapest) **A. 782.** Prove that the edges of a simple planar graph can always be oriented such that the outdegree of all vertices is at most three. (*UK competition problem*)

Problems in Physics

(see page 378)

M. 397. By measuring the temperature of a smoked metal plate (which was smoked with the flames of a candle) determine the amount of energy absorbed by a unit area surface perpendicular to the radiation emitted by the Sun in a unit of time. (Take the specific heat capacity of the metal from a table.)

G. 713. An 80-kg physics teacher makes a first class lever from a 6-m long strong wooden plank of mass 40 kg, with which he demonstrates to his students that he can lift a load of mass even up to 500 kg. Where should he place the fulcrum of the lever if he wants to lift the load up to the highest possible position such that he gently puts his total weight onto the other end of the plank? The vertical line through the centre of mass of the load is at a distance of 20 cm from the end of the plank. **G. 714.** The Earth's ice caps and glaciers currently contain approximately 30 000 000 km^3 of ice. Let us estimate how much the sea level of the oceans and seas would rise if all this huge amount of ice melted. **G. 715.** A circuit consists of three resistors and a battery as shown in the *figure.* *a*) What is the current flowing through each resistor and the voltage across them? *b*) How do these values change if we connect a lot of ("infinitely many") 1 kΩ resistors in parallel to the two resistors, already connected in parallel? *c*) What will the currents through the original three resistors and voltages across them be, if we connect a lot of ("infinitely many") 1 kΩ resistors in series with the 5 kΩ resistor? **G. 716.** A grenade fired from a cannon explodes into two pieces of equal mass at the top of its trajectory, when its speed is 100 m/s. One piece starts to move vertically upwards at a speed of 50 m/s. In what direction and at what speed does the other piece start? (The mass of the explosive in the grenade is negligible.)

P. 5240. How many litres of air is displaced from a room of sides 6 m × 5 m × 3 m, if the temperature of the air increases from 27 °C to 30 °C, while the pressure decreases by 0.5%? **P. 5241.** On May 14, 1962, due to the strong South West wind the water level of lake Balaton near city Keszthely decreased by 45 cm in 9 hours, while near the village