Adatok: $m=300$ g, $\alpha=30°$, $r=2$ cm, $\gamma=1{,}2$ Vs, $U_0=4{,}5$ V, $R=0{,}8\ \Omega$, $R_\mathrm{b}=1{,}2\ \Omega$. (A kerekek és a lejtő közötti tapadási súrlódás elég nagy, így az autó nem csúszik meg.)

a) Mekkora állandósult sebességgel halad a kisautó, ha $m=600$ g teher van a platóján?

b) Mekkora m teher esetén lesz a legjobb a szállítás hatásfoka? (A hatásfokon a teher emelésére fordított energia és a telep által leadott energia hányadosát értjük.)

(*5 pont*) Közli: *Olosz Balázs,* Pécs

P. 5239. Egy vékony, elhanyagolható tömegű, 21 cm hosszú, merev rúd végein egy-egy azonos tömegű, pontszerűnek tekinthető, kicsiny test van. Ezt a rudat a közepénél fogva felfüggesztjük egy olyan vékony, rugalmas szálra, hogy az így kapott torziós inga kis kitérések esetén mérhető lengésideje viszonylag nagy, 600 másodperc legyen. Ezután az ingát belógatjuk két, egyenként 600 kg tömegű, nagy ólomgolyó közé, középre. Az ólomgolyók középpontjai egymástól 70 cm-re vannak. Mennyi lesz az inga lengésideje kis kitérések esetén, ha az ingarúd kezdetben

a) a két golyó középpontját összekötő vízszintes szakaszon van;

b) az előbbi esetre merőleges helyzetű?

Megjegyzés. Hasonló módon határozta meg Eötvös Loránd a gravitációs állandót, két, mintegy 600 kg tömegű ólomhasáb és egy hasonlóan nagy lengésidejű torziós inga segítségével. A feladatban a gravitációs állandó ismert értékének felhasználásával kell a kétféle lengésidőt kiszámítani. (Lásd még a P. 5166. feladat megoldását a KöMaL 2020. évi márciusi számában.)

(*6 pont*) Közli: *Radnai Gyula,* Budapest

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Problems in Mathematics

New exercises for practice – competition C (see page 288): **Exercises up to grade 10: C. 1609.** Solve the following simultaneous equations over the set of real numbers: $x+y+\frac{x}{y}=19$, $\frac{x(x+y)}{y}=60$. **C. 1610.** In a unit circle, the diameter AB and the chord AC enclose a 30° angle. Let B' denote the reflection of B about the point C. Determine the distances between B and the points where the tangents drawn from B' to the circle intersect the line AB. **Exercises for everyone: C. 1611.** Some numbers are selected from the set of the first 21 positive integers such that the absolute values of the differences of all pairs of selected numbers should be different. What is the largest possible number of different absolute values obtained? Give an example of a case when this occurs. **C. 1612.** The convex heptagon $A_1A_2A_3A_4A_5A_6A_7$ has a circumscribed circle centred at an interior point of the heptagon. Prove that the sum of the interior angles at the vertices A_1, A_3 and A_5 is less than 450°. **C. 1613.** There were n teams participating in a basketball championship. Every team played every other team exactly once, and there was no draw. At the end of the championship, the ith team had x_i games won and y_i games lost ($i=1,2,\ldots,n$). Prove that $x_1^2+x_2^2+\cdots+x_n^2=y_1^2+y_2^2+\cdots+y_n^2$. (*Croatian problem*) **Exercises upwards of grade 11: C. 1614.** 12 round muffins of diameter 9 cm are arranged along the edge of a round tray of radius 30 cm such that they all touch the edge of the tray, and the neighboring muffins are separated by the same distance from each other. What is this equal distance? **C. 1615.** Julie's grandmother bakes cookies every Monday. She always selects out of her infinite number of recipes at random. 60% of her recipes contain chocolate chips. Julie is quite picky about cookies: she only likes 90% of grandma's chocolate chip cookies, and only 30% of the other kinds of cookies. On a special Monday, grandmother is making two different kinds of cookies. Find the probability that Julie will like exactly one of them.

New exercises – competition B (see page 289): **B. 5102.** There are n distinct points in the plane, which are not all collinear. Show that there exists a closed polygon with these vertices that does not cut through itself. (A polygon is allowed to have angles equal to 180°, too.) (*3 points*) **B. 5103.** Let a, b, c, x, y and z be positive numbers that satisfy the equalities $a^2+b^2=c^2$ and $x^2+y^2=z^2$. Prove that $(a+x)^2+(b+y)^2\leqslant(c+z)^2$, and determine the condition for equality. (*3 points*) (Proposed by *S. Kiss,* Nyíregyháza) **B. 5104.** Let A_1, B_1 and C_1 denote the points of tangency of the incircle of triangle ABC on the sides, and let R and r be the radii of the circumscribed and inscribed circles, respectively. Prove that the ratio of the areas of triangles $A_1B_1C_1$ and ABC is $r:2R$. (*4 points*) **B. 5105.** Let n denote a positive integer. Determine the smallest number of colours k that are sufficient for colouring the edges of any directed simple graph of n vertices without producing a circuit of the same colour. (*4 points*) (Proposed by *K. Szabó,* 11th grade student of Fazekas Mihály Primary and Secondary School and Training Centre, Budapest) **B. 5106.** The numbers $n+1,n+2,\ldots,2n$ are written on a blackboard ($n\geqslant 2$), and the following procedure is repeated: two numbers are selected (x and y) from the board, erased, and replaced with the numbers $x+y+\sqrt{x^2+y^2}$ and $x+y-\sqrt{x^2+y^2}$. Prove that there will never be a number less than 1.442 written on the board. (*5 points*) **B. 5107.** The diagonals of a cyclic quadrilateral $ABCD$ intersect at F, the lines of sides AB and CD intersect at E, the midpoint of line segment EF is G, the midpoint of line segment BF is H, and the midpoint of side BC is I. Show that $\angle GFD=\angle GIH$. (*6 points*) **B. 5108.** The points A, B_1, B_2, B_3, C_1, C_2, C_3, in this order, lie on the same line. On one side of this line, perpendicular rays b_i are drawn from the points B_i, and semicircles c_i are drawn with diameters AC_i ($i=1,2,3$), as shown in the *figure.* Prove that if the region bounded by b_1, c_1, b_2, c_2 and the region bounded by b_2, c_2, b_3, c_3 both have inscribed circles then the region bounded by b_1, c_1, b_3, c_3 also has an inscribed circle. (*5 points*) **B. 5109.** Let $x_1=2$, $x_2=7$, $x_{n+1}=4x_n-x_{n-1}$ ($n=2,3,\ldots$).

Is there a perfect square in this sequence? (*6 points*) (Proposed by *G. Stoica,* Saint John, Canada)

New problems – competition A (see page 291): **A. 777.** A finite graph $G(V,E)$ on n points is drawn in the plane. For an edge e of the graph let $x(e)$ denote the number of edges that cross over edge e. Prove that $\sum_{e\in E}\frac{1}{x(e)+1}\leqslant 3n-6$. (Submitted by: *Dömötör Pálvölgyi,* Budapest) **A. 778.** Find all square-free integers d for which there exist positive integers x, y and n satisfying $x^2+dy^2=2^n$. (Submitted by: *Kada Williams,* Cambridge) **A. 779.** Two circles are given in the plane, Ω and inside it ω. The center of ω is I. P is a point moving on Ω. The second intersection of the tangents from P to ω and circle Ω are Q and R. The second intersection of circle IQR and lines PI, PQ and PR are J, S and T, respectively. The reflection of point J across line ST is K. Prove that lines PK are concurrent.

Problems in Physics

(see page 313)

M. 396. Measure the thickness of a piece of adhesive tape.

G. 709. Tarzan targets monkey Maki on a tree. At the moment when the arrow is shot the arrow is aimed at the banana in Maki's hand. At the same moment the frightened monkey drops the banana. What will the arrow hit, if air resistance can be neglected? **G. 710.** Anna and Tom are classmates, and they live in a straight street in different houses. Every day they start walking to school at the same time at uniform speeds. Tom lives further from the school, but he is faster, in some time he overtakes Anna. One day Anna would like to meet Tom earlier, so she begins to walk towards Tom. This time they meet five times earlier than they meet usually. By what factor is Tom's speed greater than Anna's speed? **G. 711.** A closed underground chamber is connected to the outside world with a chimney. There is water in some parts of the chamber. Determine the pressure at the points A, B, C and D shown in the *figure.* **G. 712.** Statement: At not too small temperature values the molar heat capacity of metals is the same, approximately $3R=24.9$ J/(mol K), where R is the so called Regnault constant or universal gas constant. Investigate with what percentage accuracy this statement is satisfied for aluminium, gold, silver, copper and iron.

P. 5230. Laminated steel (wrongly called Damascus steel) is made by forging two layers of steel with different carbon content, which like puff pastry, is stretched to double its area and then folded in half. How many times would this process have to be repeated in order to make the thickness of a single layer atomic in size if initially the thickness of the steel was 3 mm? **P. 5231.** An apple is held at its stem on three threads of equal length. The threads are alike, they break at the same load. The upper ends of the threads are slowly separated from each other in a horizontal plane so that the angle between any two pairs of threads is the same. The threads are torn when they are (pairwise) just perpendicular to each other. If we were to attach the same apple to two of the same threads and then separate the upper ends of the threads in the same horizontal plane, what angle would the threads make with each other when they tore apart? **P. 5232.** The radius of the bottom sphere of a thin-walled, celluloid roly-poly toy is 3 cm. Inside the toy, a 2 cm diameter steel ball was fixed at the bottom. The roly-poly toy is slowly deflected such that the angle between the vertical and its axis of symmetry is 30°. What