A földi kutatók a következő információkat kapták az idegen civilizáció fizikusaitól: bolygójuk körpályán kering a csillagja körül, a pálya sugara (nevezhetjük ezt „földöntúli CSE-nek”) $\frac{1}{40\,000}$ „földöntúli fényév”. A csillagjuk tömege $2{,}4\cdot10^{57}$ földöntúli tömegegység. A $2{,}77\cdot10^{-31}$ földöntúli fényév hullámhosszúságú foton energiája éppen a földöntúli tömegegységhez tartozó nyugalmi energiával egyezik meg.

a) Hányszorosa a távoli csillag tömege a mi Napunk tömegének?

b) A „földöntúli csillagászati egység” hány földi CSE, és a távoli bolygó keringési ideje hány földi év?

c) Mekkora a földöntúli tömegegység kilogrammban kifejezve?

(Az univerzális fizikai állandók az univerzum minden részében ugyanakkorák, számértékük csak az eltérő mértékegységek miatt különbözhetnek.)

Közli: *Bertalan Zoltán,* Békéscsaba

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Problems in Mathematics

New exercises for practice – competition C (see page 225): **Exercises up to grade 10: C. 1602.** Two tenth-grade students and two eleventh-grade students sat down to solve the exercises of type C in the April issue of KöMaL*. After an hour, they observed that each exercise was solved by exactly one of them, and that everyone solved at least one exercise. In how many different arrangements may they have solved the exercises? (Two arrangements are considered different if there is at least one exercise that is solved by a different student.) **C. 1603.** The altitude drawn from vertex A of an isosceles triangle ABC intersects the leg BC at T. Let M denote the orthocentre, and let O be the centre of the inscribed circle. Prove that if line OT is parallel to the base AB, then $MC=2AM$. **Exercises for everyone: C. 1604.** A farmer brought 1225 packets of seeds to an agricultural fair: 1 packet of 1 gram of seeds, 2 packets of 2 grams, 3 packets of 3 grams, …, k packets of k grams of seeds in each – every positive integer 1 to k occurred. What was the average mass of seeds in a packet? **C. 1605.** The diagonals of a convex quadrilateral $ABCD$ intersect at M. The area of triangle ABM is greater than the area of triangle CDM. The midpoint of side BC of the quadrilateral is P, and the midpoint of side CD is Q, $AP+AQ=\sqrt{2}$. Prove that the area of quadrilateral $ABCD$ is

*There are seven exercises each month. Exercises 1–5 are for students in grade 10 at most, while exercises 3–7 may be solved by 11th and 12th grade students.

less than 1. **C. 1606.** The areas of two faces of a cuboid are 40 and 56 units. The length of the diagonal of the cuboid is $\sqrt{138}$ units of length. Calculate the possible surface area and the volume of the cuboid. (*S. Kiss,* Nyíregyháza) **Exercises upwards of grade 11: C. 1607.** Between 4 and 9, some digits of 4 are inserted, followed by the same number of digits of 8 (e.g. 4489). Prove that the resulting number is a perfect square. **C. 1608.** We are making a Vietnamese hat for a costume party. The hat is a right circular cone of apex angle 97.18°. The slant height is 28 cm. Is it possible to make a hat like this out of a 50×70 cardboard sheet available at the stationery store?

New exercises – competition B (see page 226): **B. 5094.** Prove that if two right-angled triangles have the same perimeter and the same area, then they are congruent. (*3 points*) (*S. Kiss,* Nyíregyháza) **B. 5095.** Let a, b, c denote distinct nonzero integers. Prove that if the sum of the three numbers $\frac{ab}{c}$, $\frac{bc}{a}$ and $\frac{ca}{b}$ is an integer, then each of the three numbers is an integer. (*3 points*) (*G. Stoica,* Saint John, Canada) **B. 5096.** In a regular triangle ABC of unit sides, let P be an arbitrary point on the circumference of the incircle. Let D, E, and F denote the orthogonal projections of point P on the sides BC, AC and AB, respectively. Prove that the area of triangle DEF is a constant, independent of the choice of P. (*4 points*) **B. 5097.** The product of the positive numbers $x_1, x_2, \ldots, x_n$ is 1. Prove that $x_1^4+x_2^4+\cdots+x_n^4 \geqslant x_1^3+x_2^3+\cdots+x_n^3$. (*4 points*) (*Dinu Ovidiu-Gabriel,* Bălceşti, Romania) **B. 5098.** Two players, First and Second, are playing the following game: First selects a positive integer not greater than 2020, which Second is trying to find out by guessing (by naming a number as a guess). The possible answers of First are as follows: "My number is smaller than that."; "You are right."; "My number is greater than that." If the answer is "My number is smaller than that" or "You are right", then Second pays 10 forints (Hungarian currency) to First. If the answer is "My number is greater than that" then he pays 20 forints. What is the minimum possible amount of money that Second needs to have in order to be certain that he can find out the number, and what strategy should he use? (The game terminates with the first "You are right" answer, even if Second already knows the number before asking the last question.) (*5 points*) **B. 5099.** The angle at vertex A of a rhombus $ABCD$ is 60°. An ellipse is inscribed in the rhombus, with the axes lying along the diagonals of the rhombus. The points of tangency of the ellipse on sides divide the sides in a ratio 1 : 3. On sides AB and AD it is the point closer to A, and on sides BC and CD it is the point closer to C. Let some point P move along the ellipse. Draw lines through P, parallel to the midlines of the rhombus, and consider the intersections with the other midline. Let these point be Q and R. Show that the length of the line segment QR is independent of the position of P. (*5 points*) **B. 5100.** Show that it is always possible to select some numbers (at least one) out of n consecutive integers such that their sum is divisible by $(1+2+\cdots+n)$. (*6 points*) (Based on the idea of *B. Kovács* and *Zs. Várkonyi*) **B. 5101.** $ABCDO$ is a four-sided pyramid, and P is a point in the interior of base $ABCD$. A plane not passing through O cuts the lines OA, OB, OC, OD and OP at points A', B', C', D', and P', respectively. Prove that $\frac{t_{PAB}\cdot t_{PCD}}{t_{PBC}\cdot t_{PDA}}=\frac{t_{P'A'B'}\cdot t_{P'C'D'}}{t_{P'B'C'}\cdot t_{P'D'A'}}$, where t_{XYZ} denotes the area of triangle XYZ. (*6 points*)

New problems – competition A (see page 228): **A. 775.** Let $H\subseteq\mathbb{R}^3$ such that if we reflect any point in H across another point of H, the resulting point is also in H. Prove that either H is dense in $\mathbb{R}^3$ or one can find equidistant parallel planes which cover H. (Submitted by *Árpád Kurusa,* Szeged and *Vilmos Totik,* Szeged) **A. 776.** Let

$k>1$ be a fixed odd number, and for non-negative integers n let $f_n=\sum_{\substack{0\leqslant i\leqslant n\\ k|n-2i}}\binom{n}{i}$. Prove that f_n satisfy the following recursion: $f_n^2=\sum_{i=0}^{n}\binom{n}{i}f_if_{n-i}$. (Submitted by *András Imolay,* Budapest)

Problems in Physics

(see page 249)

M. 395. Measure the rate of airflow of a hair dryer at different speed settings.

G. 705. Two balls are released from an airship floating at a great height. Which ball falls faster, if *a*) they have the same size, but different weights; *b*) they have the same weight, but different sizes? **G. 706.** The docudrama titled Apollo 13 is about the happily ended accident of the spacecraft, occurred in 1970. The scenes depicting weightlessness were recorded in a Boeing KC-135 air-plane (belonging to NASA) in 612 short, 23 second-long parts. In order to create the "zero gravity" for each of these short parts the plane flew along a parabolic path. What was the smallest speed of the plane during the zero gravity period of the flights if the angle between the horizontal and tangent to the parabolic path of the plane both at he beginning and at the end of the zero gravity manoeuvre was 45°? **G. 707.** Sam and Sarah are jogging along a straight path at constant speed. Sam's speed is 3 m/s and Sara's speed is 2 m/s. Pluto, their dog, is running back and forth between them. Initially Sarah is 20 m ahead Sam. Pluto is a "miraculous" dog, because he can run between them at a constant speed of 4 m/s such that all his turns are immediate. Considering Pluto's arbitrary initial location and direction of running, determine the smallest and greatest values of both the distance covered and the displacement of the dog until Sam and Sarah meet. **G. 708.** Ben ran into a mirrored labyrinth of an amusement park, and hid at point B. Can his mother, who is looking for Ben standing at point A, see him? The plan of the mirrored labyrinth of the amusement park is shown in the *figure.* The thick lines represent mirrors with reflexive surfaces at both of their sides.

P. 5219. There is a shadoof in the middle of a meadow in a plain. Its vertical pole has half the length of its horizontal beam. Reaching the rim of the meadow, the shadoof is at a distance of 100 m from us towards the north. Observing the shadoof from the rim of the meadow the angle subtended by its vertical pole is 2.3°. Our eyes are at a height of 165 cm, the horizontal beam is east–west and is supported by the pole at its centre. We walk towards the shadoof at a constant speed of 1 m/s. Calculate and sketch how the angle subtended by the shadoof change over time as we walk from the rim of the meadow to the shadoof until we reach it. **P. 5220.** A compressed spring, which is tied with a piece of thread, is placed between two carts of masses M and $2M$ such that it is fixed to only one of the carts. This system is placed to a horizontal frictionless tabletop and given an initial speed of v_0. After some time the thread breaks, and because of this one of the carts stops. *a*) At what speed does the other cart move further? *b*) How much energy was stored in the spring? **P. 5221.** A frictionless track is built for a small (point-like) toy car. The initial part of the track is horizontal, and then it continues in a vertical circular loop of radius r, and horizontal again after the loop is closed. Let v be the least initial speed of the toy car at which it must be started in order that it just go along the circular loop. What fraction of this speed v should be given to the toy car in order that after leaving the loop it strike the track exactly at the opposite point of the circle? **P. 5222.** Two solid rubber balls, made of high-quality rubber, are placed on the top of one another as shown in the *figure,* and then they are released from a height of h. The collisions occur as follows: first the bottom ball of mass M collides with the ground totally elastically, then after a very short time the ball that bounced back from the ground collides totally