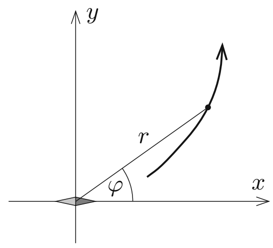

P. 5218. A derékszögű koordináta-rendszer origójában elhelyezett kicsiny, „pontszerű” mágnestű az x tengely irányába mutat. Egyik mágneses erővonalának egyenlete $r = r_0 \sin^2 \varphi$, ahol r és φ az erővonal egy-egy pontjának ún. polárkoordinátái.

a) Írjuk fel ennek az erővonalnak az egyenletét x és y koordinátákkal kifejezve, ha $r_0 = 3$ méter!

b) Az erővonalnak hol vannak olyan pontjai, ahol a mágneses indukcióvektor iránya merőleges a mágnestűre?

(*6 pont*) Közli: *Radnai Gyula,* Budapest

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Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
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Problems in Mathematics

New exercises for practice – competition K (see page 159): **K. 654.** There were 20 people at a meeting. It turned out that everyone knew exactly 13 of the other participants (acquaintance is mutual). What is the minimum possible number of acquaintances that an arbitrary pair of participants may have in common? **K. 655.** The four-digit numbers $\overline{ABCD}$, $\overline{BCBA}$, $\overline{BDAB}$ and $\overline{DDAD}$ are distinct four-digit primes (different letters denote different digits). Which numbers are they? You can use website `http://matek.com/szamok/primszamok` to check if a particular four-digit number is a prime number. **K. 656.** Given a 21 cm by 29 cm rectangular sheet of paper, how can you use it to measure a distance of *a*) exactly 3 cm, *b*) exactly 1 cm, without using anything else? (It is allowed to fold the sheet of paper.) **K. 657.** Find all multiples of 99 from 1 to 10 000 in which the sum of the digits is not divisible by 18. **K. 658.** In each of two rectangular rooms of the same floor area, the floor is covered with 25 cm × 40 cm tiles. No tile is cut. In one room, the 40-cm sides of the tiles are parallel to the longer side of the rectangle, and in the other room they are parallel to the shorter side. In one room, there are 9 fewer tiles along the longer wall than in the other room, and 6 more tiles along the shorter wall than in the other. How long are the sides of the bases of the rooms?

New exercises for practice – competition C (see page 160): **Exercises up to grade 10: C. 1595.** Find all pairs (x, y) of positive integers such that $\frac{1}{x} + \frac{1}{y} = \frac{2}{1893}$. (Could have been proposed by *Zaphenath Paaneah,* Thebes, Egypt) **C. 1596.** The sides of a triangle are 5 cm, 5 cm and 6 cm long. The sides, and the tangents drawn to the incircle parallel to the sides form a hexagon. What is the area of this hexagon? **Exercises for everyone: C. 1597.** How many different right-angled triangles are there in which the measures of the sides are integers, and one side is 2^n units long? (Where n is a positive integer: express your answer in terms of n.) **C. 1598.** The length of the line segment MN joining the midpoints of sides AB and CD in a convex quadrilateral $ABCD$ is the arithmetic mean of the lengths of sides AD and BC. Show that the quadrilateral $ABCD$ is a trapezium. **C. 1599.** Solve the following equation over the set of pairs of natural numbers: $2y^2-2x^2-3xy+3x+y=13$. (Proposed by *T. Imre,* Marosvásárhely) **Exercises upwards of grade 11: C. 1600.** Solve the following equation over the set of real numbers: $4^x+9^x+36^x+\sqrt{\frac{1}{2}-2x^2}=1$. (Proposed by *B. Bíró,* Eger) **C. 1601.** The height of a lateral face of a right pyramid with a square base is twice as long as the base edge. At what percentage of this height (counting from the base) do we need to cut the pyramid with a plane parallel to the base so that the total area of the lateral surface plus top square of the resulting frustum is equal to half the lateral surface area of the original pyramid?

New exercises – competition B (see page 161): **B. 5086.** Solve the equation $\left(x^3-y^2\right)^2=\left(x^2-y^3\right)^2$ over the set of pairs of integers. (*4 points*) (Proposed by *M. Szalai,* Szeged) **B. 5087.** The distances of an interior point P of a square $ABCD$ from the vertices A, B, D are 1, $\sqrt{2}$, and 2, respectively. Calculate the measure of the angle APB. (*4 points*) (Proposed by *B. Bíró,* Eger) **B. 5088.** With respect to a given set G of numbers, the positive integer $k>1$ is called *interesting* if there exist k distinct elements in set G such that the arithmetic mean of these elements also belongs to set G. Let $H=\{1;3;4;9;10;\ldots\}$ be the set of those numbers that can be represented as a sum of some different powers of 3. *a*) What numbers $k>1$ are interesting with respect to the set H? *b*) Let $c\notin H$ be an arbitrary positive integer. Prove that every number $k>1$ is interesting with respect to the set $H'=H\cup\{c\}$. (*5 points*) **B. 5089.** Two skew edges of a tetrahedron are perpendicular to each other, their lengths are 12 and 13, and the distance between their lines is 14 units. Determine the volume of the tetrahedron. (*3 points*) **B. 5090.** The inscription on one side of a fair coin is $+1$, and -1 is on the other side. The coin is tossed n times in a row, and the n results are written down in a row. Then the product of every pair of consecutive items is written below them, resulting in a new list of numbers that only consists of $(n-1)$ items. The procedure is repeated until a single number remains. What is the expected value of the sum of the $\frac{n(n+1)}{2}$ numbers written down in the resulting triangular arrangement of numbers? (*3 points*) **B. 5091.** In a regular dodecagon $A_1A_2\ldots A_{12}$, let P denote the intersection of the diagonals A_1A_8 and A_6A_{11}, and let R denote the intersection of lines A_7A_8 and A_9A_{11}. Show that the line PR divides diagonal A_1A_4 in a $2:1$ ratio. (*5 points*) (Based on the idea of *B. Bíró,* Eger) **B. 5092.** The sum of the elements is calculated for each subset of the set $\{0,1,\ldots,n-1\}$. What may the value of n be if exactly one nth of the resulting 2^n sums is divisible by n? **B. 5093.** The intersection of two congruent regular pentagons is a decagon with sides of $a_1,a_2,\ldots,a_{10}$ in this order. Prove that $a_1a_3+a_3a_5+a_5a_7+a_7a_9+a_9a_1=a_2a_4+a_4a_6+a_6a_8+a_8a_{10}+a_{10}a_2$.

New problems – competition A (see page 162): **A. 772.** Each of N people chooses a random integer number between 1 and 19 (including 1 and 19, and not necessarily with the same distribution). The random numbers chosen by the people are independent from each other, and it is true that each person chooses each of the 19 numbers with probability at most 99%. They add up the N chosen numbers, and take the remainder of the sum divided by 19. Prove that the distribution of the result tends to the uniform distribution exponentially, i.e. there exists a number $0<c<1$ such that the mod 19 remainder of the sum of the N chosen numbers equals each of the mod 19 remainders with probability between $1/19-c^N$ and $1/19+c^N$. (Submitted by *Dávid Matolcsi,* Budapest) **A. 773.** Let $b\geqslant 3$ be a positive integer and let σ be a nonidentity permutation of the set $\{0,1,\ldots,b-1\}$ such that $\sigma(0)=0$. The *substitution cipher* C_σ encrypts every positive integer n by replacing each digit a in the representation of n in base b with $\sigma(a)$. Let d

be any positive integer such that b does not divide d. We say that C_σ *complies* with d if C_σ maps every multiple of d onto a multiple of d, and we say that d is *cryptic* if there is some C_σ such that C_σ complies with d. Let k be any positive integer, and let $p = 2^k + 1$. a) Find the greatest power of 2 that is cryptic in base $2p$, and prove that there is only one substitution cipher that complies with it. b) Find the greatest power of p that is cryptic in base $2p$, and prove that there is only one substitution cipher that complies with it. c) Suppose, furthermore, that p is a prime number. Find the greatest cryptic positive integer in base $2p$, and prove that there is only one substitution cipher that complies with it. (Submitted by *Nikolai Beluhov,* Bulgaria) **A. 774.** Let O be the circumcenter of triangle ABC, and D be an arbitrary point on the circumcircle of ABC. Let points X, Y and Z be the orthogonal projections of point D onto lines OA, OB and OC, respectively. Prove that the incenter of triangle XYZ is on the Simson-Wallace line of triangle ABC corresponding to point D. (Submitted by *Lajos Fonyó,* Keszthely)

Problems in Physics

(see page 185)

M. 394. Make an approximately 80 cm long paper strip, and fix its ends at the same height to some moveable stands. Place a small cylinder-shaped tin can on the middle of the strip such that the tin pulls down the strip a little. Then displace the tin always by the same amount (approximately 20 cm), and then release it without any initial speed, and let it roll freely. Measure the period of the (damped) periodic motion of the tin, as a function of the sag of the strip h in the case of a) a full tin can; b) an empty tin can.

G. 701. What is the ratio of the number of revolutions of the two pulleys shown in the *figure,* if their radius is the same? (The threads between the pulleys can be considered vertical.) **G. 702.** A disc-shaped permanent magnet of a mass of 80 g clings to a vertical iron sheet. The flat disc can be slid vertically downwards by applying a force of 2 N. With what force can the magnet be moved upwards? What is the magnitude and the direction of the force which should be applied in order to move the disc horizontally? (The applied force is parallel to the plane of the sheet in all cases.) **G. 703.** How can we determine the internal resistance of a durable battery by using a digital voltmeter (which can be considered ideal) and a resistor of known resistance? (Wires can also be used.) **G. 704.** When Torricelli's experiment is carried out at sea level, then the mercury column is 76 cm high. However, on a very high hill the height of the mercury is only 40 cm. How high is the hill?

P. 5208. A basketball of mass 0.6 kg bounces to a height of 0.57 m when it is dropped form a height of 1.05 m. a) What is the mechanical energy loss due to the collision with the ground? b) What is the ratio of the speed at which the basketball bounced back to the speed of the ball when it reached the ground? (This ratio is called the coefficient of restitution.) c) In order to compensate the energy loss, players usually dribble the basketball, during which they push the ball downwards for a short while. Suppose a player pushes the ball along a distance of 0.08 m, starting it at a height of 1.05 m. What is the average force exerted by the player if the ball bounces back to a height of 1.05 m? **P. 5209.** In the pulley system shown in the *figure* the fixed pulley at the top has a radius of 15 cm, whilst the radius of the moveable pulley at the bottom is 25 cm. Each of the moveable pulleys turns 15 whole revolutions in a minute, and the rotational speeds of the fixed pulleys are also equal. (The threads between the pulleys can be considered vertical.) a) What is the radius of each of the other pulleys? b) What is the number of revolutions of the fixed pulleys? **P. 5210.** The crew of the spacecraft Apollo 11 (Neil Armstrong, Edwin Aldrin és Michael Collins) performed the first landing on the Moon. The spacecraft was