a) Mekkora a müonok (átlagos) sebessége és mozgási energiája, ha a tároló-gyűrű átmérője 120 m?

b) Mekkora a gyűrűben a mágneses indukció nagysága?

(*6 pont*)   Közli: *Fajszi Bulcsú,* Budapesti Fazekas M. Gyak. Ált. Isk. és Gimn.

Beküldési határidő: 2020. március 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 70. No. 2. February 2020)

Problems in Mathematics

New exercises for practice − competition K (see page 95): **K. 649.** A fast train and a slow train of the same length are travelling on two parallel tracks, in opposite directions. The tracks both pass through a tunnel. The fronts of the two trains arrive at the two entrances of the tunnel simultaneously. It takes 3 seconds for the total length of the fast train to become covered by the tunnel, and it takes 6 seconds for the slow train. The trains meet inside the tunnel 18 seconds after reaching the tunnel. How long do they take to pass each other? At what time after meeting will the full length of the individual trains emerge from the tunnel? **K. 650.** The side of the small square in the *figure* is 3 cm, the sides of the large rectangle have integer lengths, one being 2 cm longer than the other. The sides of the rectangle and the square are parallel, their centres coincide. The shaded region was made by extending the sides of the small square in one direction, and connecting the points where the sides of the large rectangle were reached. Is it possible for the area of the shaded region to be an even number (of cm^2)? **K. 651.** For the areas marked in the *figure,* $T_1 : T_2 : T_3 = 2 : 7 : 3$. What are the ratios of the lengths x to y, and u to v? **K. 652.** A box contains yellow, blue and red balls, 10 of each colour. In how many different ways is it possible to divide the balls into a group of 10 and a group of 20 such that each group should contain at least one of each colour? (Balls of the same colour cannot be distinguished.) **K. 653.** We know that $\sqrt{a\sqrt{a\sqrt{a}}} = b$ and $a, b > 1$ are integers. Find the minimum value of $a + b$.

New exercises for practice − competition C (see page 96): **Exercises up to grade 10: C. 1588.** Let E and F be the points lying closer to vertex A which divide the sides AB and AD of a quadrilateral $ABCD$, respectively, in a $1 : 2$ ratio. Let G be the point lying closer to B which divides side BC in a $1 : 2$ ratio. Reflect point G in the point E, and reflect its image in the point F. Prove that the final image lies on a side of the quadrilateral. Which side is it, and in what ratio is it divided by the final image? **C. 1589.** Solve the following equation over the set of real numbers: $(y^2 + y - x - 1)^2 + \left(x + \frac{1}{x}\right)^2 = 4$. (Proposed by *B. Bíró,* Eger) **Exercises for everyone: C. 1590.** Find the positive integer solutions of the equation $(a + 1)^4 \cdot (b + 1)^4 \cdot (c + 1)^4 = (40a + 1) \cdot (40b + 1) \cdot (40c + 1)$. **C. 1591.** The coordinates of a ship are $x = 2$, $y = 0$. The shoreline is given by the curve of equation $y = \sqrt{2x + 1}$. At what angle should the ship deviate from the direction due north in order to reach the closest point of the shore in a straight line? (Assume that the x-axis points towards the east.) **C. 1592.** In England,

a man lost his wedding ring, and set out to search for it with the help of a friend and a metal detector. They did not find the ring, but they did find some gold coins from the time of King Henry VIII, delivering 100 000 pounds to the two friends. The one-pound coins were preserved in very good condition, and during the 500 years elapsed, their average annual increase in value was between 1.42% and 1.43%. How many coins may they have found? **Exercises upwards of grade 11: C. 1593.** Two sides of a triangle are 3 cm and 4 cm long. What is the angle enclosed by the sides if the medians drawn from the opposite vertices are perpendicular to each other? **C. 1594.** The first row of seats in an auditorium consists of 24 seats. 20 seats are already taken. What is the probability that there are 2 adjacent vacant seats?

New exercises – competition B (see page 97): **B. 5078.** Define a sequence $a_1, a_2, \ldots$ by the following recurrence relation: $a_1 = 1$, $a_n = \frac{n+1}{n-1}(a_1 + a_2 + \cdots + a_{n-1})$ for $n > 1$. Determine the value of a_{2020}. (*4 points*) **B. 5079.** Solve the equation $\log_2 \log_3 x + \log_3 \log_2 x = \log_2 \frac{6}{\log_2 3}$ over the set of real numbers. (*3 points*) (Proposed by *B. Bíró*, Eger) **B. 5080.** Let D denote the midpoint of the base AB in an isosceles triangle ABC. Let H be the point lying closer to C that divides the leg AC in a $1 : 2$ ratio. The circle BCH intersects line CD at the points C and X. Show that $CX = \frac{4}{3}r$, where r is the radius of the circle ABC. (*4 points*) **B. 5081.** In a triangle, the medians drawn to sides a and b are perpendicular. Prove that $\frac{1}{2} < \frac{a}{b} < 2$. (*3 points*) **B. 5082.** Prove that the geometric mean, the arithmetic mean, and the quadratic mean of the altitudes in any triangle is not greater than the geometric mean, the arithmetic mean, and the quadratic mean of the radii of the escribed circles, respectively. (*5 points*) **B. 5083.** Is there a polynomial $p(x)$ of degree 100 with real coefficients for which the polynomial $p(p(x))$ has 10000 distinct real roots? (*5 points*) **B. 5084.** Let n be a positive integer, and let $\mathcal{S}$ denote the set of $0 - 1 - 2$ sequences of length n. Determine which sets $\emptyset \neq A \subseteq \mathcal{S}$ have the following property: no matter how a vector $(c_1, c_2, \ldots, c_n) \in \mathcal{S} \setminus \{(0, 0, \ldots, 0)\}$ is selected, the probabilities that the sum of products $c_1 a_1 + c_2 a_2 + \cdots + c_n a_n$ formed with a randomly chosen element $(a_1, a_2, \ldots, a_n)$ of set A will leave a remainder of 0, 1, or 2 are $1/3$ each. (*6 points*) (Based on a problem of *Kürschák competition*) **B. 5085.** Show that a regular heptagon can be dissected into a finite number of symmetrical trapezoids, all similar to each other. (*6 points*) (Proposed by *M. Laczkovich,* Budapest)

New problems – competition A (see page 99): **A. 769.** Find all triples (a, b, c) of distinct positive integers so that there exists a subset S of the positive integers for which for all positive integers n exactly one element of the triple (an, bn, cn) is in S. (Proposed by *Carl Schildkraut,* Massachussets Institute of Technology) **A. 770.** Find all positive integers n such that $n!$ can be written as the product of two Fibonacci numbers. **A. 771.** Let ω denote the incircle of triangle ABC, which is tangent to side BC at point D. Let G denote the second intersection of line AD and circle ω. The tangent to ω at point G intersects sides AB and AC at points E and F. The circumscribed circle of DEF intersects ω at points D and M. The circumscribed circle of BCG intersects ω at point G and N. Prove that lines AD and MN are parallel. (Proposed by *Ágoston Győrffy,* Remeteszőlős)

Problems in Physics
(see page 121)

M. 393. Hang a piece of twisted rope or yarn, and then attach different weights to its free end. Leave the rope unwind, continuously slowing the motion with your hand until it reaches the equilibrium position. How does the angle turned by the lower end of the rope depend on the load?