Útmutatás: felhasználhatjuk, hogy

$$\int_{x_1}^{x_2} \frac{1}{x}\,\mathrm{d}x = \ln\frac{x_2}{x_1}.$$

c) Mekkora az M tömegű rakéta és a kiáramlott gázok összes mozgási energiája az indulási vonatkoztatási rendszerben?

Útmutatás: felhasználhatjuk, hogy a kiáramlott gázokból és a rakétából álló teljes rendszer mozgási energiájának megváltozása független a vonatkoztatási rendszertől, így pl. a rakétával együtt mozgó rendszerben is ugyanakkora, mint az indulási vonatkoztatási rendszerben.

d) Legfeljebb mekkora lehet a rakétameghajtás „mechanikai hatásfoka”, vagyis a rakéta mozgási energiájának és az összes mozgási energiának a hányadosa az indítási vonatkoztatási rendszerben?

(*6 pont*) *Némedi István* (1932–1998) feladata

❄

Beküldési határidő: 2020. február 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

❄

MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 70. No. 1. January 2020)

Problems in Mathematics

New exercises for practice – competition K (see page 28): **K. 644.** A box contained a total of 70 blue and green cubes. Four times as many blue cubes were removed as green cubes, so there remained 7 times as many green cubes as blue cubes in the box. How many cubes of each colour were there originally? **K. 645.** What will be the remainder if $1+4+7+\cdots+2020$ is divided by 8? **K. 646.** We have three machines. Each of them has two input channels and one output channel. As represented in the *diagram,* the machines carry out some well defined sequence of operations with the numbers obtained through the input channels, and display the final result as output. Machine A displays $x\cdot y$, machine B displays x^2+y, and machine C displays $5\cdot x+3\cdot y$ where x and y stand for the first and second input numbers, respectively. The machines A, B and C are connected by attaching the outputs of two machines to the inputs of the third one. What is the largest possible output that may be obtained from the third machine if the inputs of the first two machines are the same, $x=4$ and $y=7$? **K. 647.** An icosahedron made of paper is cut along some edges to unfold in the plane and obtain the net of the solid. How many edges need to be cut? **K. 648.** The sides of a square are divided into three equal parts. An interior point of the square is connected to one of the dividing points on each side, as shown in the *figure,* to form four quadrilaterals. Given the area of one quadrilateral (see figure), determine the areas of the other quadrilaterals.

New exercises for practice – competition C (see page 29): **Exercises up to grade 10: C. 1581.** Find all positive integers n for which $n!$ ends in exactly 19 531 zeros. **C. 1582.** Let ABP, BCQ, CDR, DAS denote the regular triangles drawn on the sides of paralellogram $ABCD$ on the outside. What requirements does the parallelogram need to meet in order for $PQRS$ to be a square? **Exercises for everyone: C. 1583.** In the Cartesian coordinate plane, represent the points for which the following inequality holds: $|x|+|y|+|x+y|\leqslant 2$. What is the area of the resulting figure? (*Croatian problem*) **C. 1584.** What is the maximum area of a square that can be obtained from a triangular sheet of paper with sides 3 cm, 4 cm and 5 cm by cutting along at most three straight lines? **C. 1585.** What distinct positive primes p and q satisfy the equality $p-4p^2+p^3=q-4q^2+q^3$? **Exercises upwards of grade 11: C. 1586.** The points D and E divide side AB of triangle ABC into three equal parts. Let P be an arbitrary interior point of line segment DE. Draw parallels to line PC through the points D and E. These lines intersect sides AC and BC at points Q and R, respectively. Show that the area of quadrilateral $PRCQ$ equals the area of triangle APQ. (Proposed by *B. Bíró,* Eger) **C. 1587.** Solve the equation $\frac{x-1}{\sqrt{x}}-\frac{\sqrt{x}}{x-2}=\frac{\sqrt{x}}{2x-1}$ over the set of real numbers. (Proposed by *B. Bíró,* Eger)

New exercises – competition B (see page 31): **B. 5070.** There are two kinds of people living on an island: some always tell the truth, while some always lie. Ten inhabitants of the island were numbered as $1,2,\ldots,10$. Then everyone was asked the same three questions: "Is your number even?", "Is your number divisible by 4?", "Is your number divisible by 5?". The number of those saying "Yes" was three, six, and two, respectively in the three cases. Which numbers were assigned to liars? (*4 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5071.** Let us consider a triangle ABC. Let A_1 and A_2 denote the points that divide side BC into three equal parts, with A_1 lying closer to vertex B. Analogously, let B_1, B_2 divide side CA into equal parts, with B_1 lying closer to C, and let C_1 és C_2 divide AB into three equal parts, with C_1 lying closer to B. Prove that the triangles $A_1B_1C_1$ and $B_2C_2A_2$ are congruent, and both of their areas are equal to the third of the area of the triangle ABC. (*3 points*) (Proposed by *B. Bíró,* Eger) **B. 5072.** Prove that $\left[\sqrt{n}+\sqrt{n+3}\right]=\left[\sqrt{4n+5}\right]$ for all positive integers n. (*3 points*) (Proposed by *T. Imre,* Marosvásárhely) **B. 5073.** The tangents drawn to the incircle of triangle ABC, parallel to the sides, cut off three small triangles at the corners. The radii of the incircles of the small triangles are 2, 3 and 10 units long. Show that triangle ABC is right-angled. (*4 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5074.** Find all positive integers n and distinct (positive) primes p, q, r for which $\frac{1}{pq}+\frac{1}{pr^3}+\frac{1}{qr^2}=\frac{1}{n}$. (*5 points*) (Proposed by *G. Holló,* Budapest) **B. 5075.** The midpoints of sides AD and BC of a convex quadrilateral $ABCD$ are E and F, respectively. Line segment EF intersects diagonal AC at point P, and diagonal BD at point Q. Prove that the circles AEP and BFQ intersect each other on the line AB. (*5 points*) (Proposed by *G. Holló,* Budapest) **B. 5076.** Find the real solutions of the system $x+y+z+v=0$, $x^2+y^2+z^2+v^2=12$, $x^3+y^3+z^3+v^3=24$. (*6 points*) **B. 5077.** We want to draw a perspective image of a cube, using two vanishing points: $I_1=(-9;0)$ and $I_2=(10;0)$, as shown in the *figure.* The images of three vertices are $A=(-3;0)$, $B=(0;0)$ és $C=(4;0)$. What should the y-coordinate of point F be? (*6 points*)

New problems – competition A (see page 32): **A. 767.** In an $n\times n$ array all the fields are colored with a different color. In one move one can choose a row, move all the fields one place to the right, and move the last field (from the right) to the leftmost field of the row; or one can choose a column, move all the fields one place downwards, and move the field at the bottom of the column to the top field of the same column. For what values of n is it possible to reach any arrangement of the n fields using these kinds of steps? (Proposed by *Ádám Schweitzer*) **A. 768.** Let S be a shape in the plane which is obtained as a union of finitely many unit squares. Prove that the ratio of the perimeter and the area of S is at most 8.

Problems in Physics

(see page 57)

M. 392. Measure the specific heat capacity of the material of a 100-forint (or 1-euro) coin in a calorimeter (thermos flask).

G. 693. Two exactly alike trains move towards each other (not necessarily at the same speed) along parallel railroad tracks next to each other. The length of each locomotive is the same as the length of each railway carriage. Both trains consist of 19 carriages and locomotives which haul the trains (one locomotive in front of each train). Peggy is travelling in the third carriage (counted from the locomotive). After the two trains meet, 36 seconds elapse when Peggy's carriage gets exactly next to that carriage of the other train in which Daniel is travelling, and then 44 seconds elapse until the two trains go past each other. Counted from the locomotive, in which carriage is Daniel? **G. 694.** An exactly 100-kg rocket, moving in space, ejects 100 g exhaust gas in each second. The exhaust leaves the nozzle at a speed of 1 km/s. What is the acceleration of the rocket? **G. 695.** Toy penguins were suspended to an unsteady room decoration. The very light rods (see the *figure*) were suspended at their quadrisection points such that the structure is in equilibrium. What are the masses of the second, the third, and the fourth toy penguins, if the first one has a mass of 480 g? **G. 696.** Alexander got a soldering iron as a Christmas present, and he immediately tried it. He soldered two resistors of resistances 2000 Ω and 500 Ω in parallel, and then with this he soldered another 500 Ω resistor in series. Finally with this he soldered a 600 Ω resistor in parallel. He connected the circuit to a battery and measured the voltage across the 2000 Ω resistor, which was 2 V. *a*) Draw a schematic figure of the circuit. *b*) What is the voltage across the battery? *c*) What is the current through the battery?

P. 5186. The total mass of a sleigh and a child on it is 25 kg. The coefficient of kinetic friction between the sleigh and the snow is 0.05. *a*) We would like to pull the sleigh horizontally at a constant speed. What is the horizontal force that must be applied? *b*) The sleigh is accelerated from rest for 2 seconds with a constant force of 50 N along the horizontal, snow covered ground, and then it is released. What is the distance covered by the sleigh from the starting position until it again comes to rest? **P. 5187.** Two simple pendulums are fixed at the same height. They swing in parallel planes close to each other. One of them is $\ell_1 = 25$ cm, whilst the other is $\ell_2 = 1.2$ m long. The two pendulums are displaced in the opposite direction at the same small angle, and then released. *a*) How long does it take for the pendulums to go past each other after they were released? *b*) How much time elapses until they meet again? *c*) What should the ratio of the lengths of the pendulums be in order that the fifth encounter be the first one when the direction of the velocities of the two pendulum bobs is the same? **P. 5188.** A cuboid-shaped prism is sliding along a horizontal surface in a straight line and is decelerating due to friction. The *figure* shows the velocity-time graph of its motion. A spring-loaded toy cannon is attached to the top of the prism, which fires projectiles of mass m at a speed of v_0, during the slowing motion of the prism. The total mass of the prism and the toy cannon is M. *a*) To which direction should the cannon be aimed in order that the projection not affect the decelerating motion of the prism, or in other words the graph of the motion of the cannon continue in the same way after firing the projectile? *b*) If the angle between the gun barrel and the horizontal is adjusted to half of the angle determined in the previous problem,