P. 5184. Egy nagy felbontású optikai rács a merőlegesen ráeső lézersugarat már első rendben 45°-os szögben képes eltéríteni. Mi történik, ha az eltérített lézersugár útjába egy másik, ugyanilyen optikai rácsot helyezünk

a) az eredeti ráccsal párhuzamosan;

b) az eredeti rácsra merőlegesen?

(A két rács rései mindkét esetben párhuzamosak egymással.)

(*5 pont*) Közli: *Radnai Gyula,* Budapest

P. 5185. Egy vízszintes lapon mozgó kis korongra a pillanatnyi sebességével arányos fékezőerő hat. Kétféle kísérletet végzünk vele:

(*i*) Ha meglökjük v_0 sebességgel, akkor a megállásáig 50 cm utat tesz meg.

(*ii*) Amikor a meglökött korong sebessége már $v_0/2$-re csökkent, nekiütközik egy másik, álló korongnak, amelyre ugyancsak a sebességével arányos fékezőerő hat. (Az arányossági tényező mindkét korongnál ugyanakkora.) Az ütközés egyenes és rugalmas. Meglepő módon a két korong egymás mellett áll meg.

a) Mekkora a két korong tömegének aránya?

b) Az ütközés helyétől milyen messze áll meg a két korong?

(*6 pont*) A *Kvant* nyomán

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Problems in Mathematics

New exercises for practice – competition K (see page 542): **K. 639.** There are 53 passengers in a bus: men, women, girls and boys. The number of women is three times the number of boys, and 10 more than the number of girls. The total number of men and boys is 15. How many men, women, girls and boys are travelling in the bus? **K. 640.** The square of a two-digit number ending in 5 can also be calculated as follows: the digit in the tens' place is multiplied by the number greater by 1, and 25 is written after the product. Explain why this method works. **K. 641.** Some points are selected in the interior of a convex quadrilateral. These points are connected to each other and to the vertices of the quadrilateral with line segments such that the line segments have no intersection inside the quadrilateral, and they divide the quadrilateral into small triangles and pentagons. (Every interior point is a vertex of some triangle or quadrilateral.) Is it possible to divide the quadrilateral into exactly 2019 polygons in this way? **K. 642.** Find all positive integers x and y such that $x^2-y^2=2019$. **K. 643.** In the fraction $\frac{a6bc}{de3fg}$

each digit, except 0, occurs exactly once. What may each letter denote if the value of the fraction is $\frac{1}{2}$?

New exercises for practice – competition C (see page 543): **Exercises up to grade 10: C. 1574.** The points marked in the *figure* are labelled with the integers 0 to 10. Then the sum of the numbers on the vertices is written in each triangular region. What are the largest and the smallest possible values of the sum of the 14 numbers obtained in this way? **C. 1575.** Find all pairs of positive primes p, q for which $2pq+2p-q=q^2-8$. (Proposed by *T. Imre,* Marosvásárhely) **Exercises for everyone: C. 1576.** A unit circle is centred at O, and P is a point such that $OP=2$. Consider a secant through P that intersects the circle at points M and N such that M bisects the line segment NP. Prove that the area of triangle OMN is smaller than $\frac{1}{2}$. **C. 1577.** The two and three digit numbers $\overline{ab}$, $\overline{abc}$, $\overline{cab}$ (in decimal notation), in this order are three consecutive terms of an increasing, infinite arithmetic sequence. How many terms does this sequence have between 1552 and 2020? (Proposed by *B. Bíró,* Eger) **C. 1578.** The circumferences of two congruent rectangles intersect at eight points. Show that the area of the overlapping part of the rectangles is greater than half the area of a rectangle. **Exercises upwards of grade 11: C. 1579.** Find the real solutions of the equation $(x-11)^{\log_2(x-10)}=(x-11)^{\log_{\frac{1}{2}}(x-11)}$. (Proposed by *B. Bíró,* Eger) **C. 1580.** Barbara places 10 coins on a table in a row at random. In each step, she turns over two adjacent coins. What is the probability that she cannot achieve after a sufficient number of steps, that all coins should have "heads" on top?

New exercises – competition B (see page 544): **B. 5062.** Solve the following simultaneous equations: $x[x]+y[y]=1$, $[x]+[y]=1$. (*3 points*) (*MI&Q*) **B. 5063.** In a triangle ABC, $BC<AC$ and $\angle ACB$ is a right angle. The tangents drawn from point A to the circle of diameter BC touch it at C and D. The line of tangent AD intersects line BC at point E. The midpoint of line segment BC is O. Prove that the area of triangle DEO equals the area of triangle AEB. (*3 points*) (Proposed by *B. Bíró,* Eger) **B. 5064.** The "board" in the *figure* consists of 26 fields. In how many different ways is it possible to cover the board with 13 "dominoes"? Each domino covers two adjacent fields. (Solutions obtained from each other by rotation are considered different.) (*4 points*) **B. 5065.** The centre of the circumscribed circle of an acute angled triangle ABC is O, and the reflections of O in sides BC, CA and AB are O_A, O_B, and O_C, respectively. Show that the lines AO_A, BO_B and CO_C are concurrent. (*4 points*) **B. 5066.** Thirty students were taking an exam in a subject called "Tautologics". The students were sitting down in a classroom, and the teacher asked them a single question, "How many of the students sitting in this room are going to fail this exam altogether?" The students had to name a number, one by one, in a row. After each answer, the teacher immediately announced the result which was either "pass" or "fail". When the exam was over, the Student Union requested an inspection by the School District. If it turns out that any student gave the correct answer but still failed the exam, all the results will be cancelled, and everyone will receive a "pass" grade. Is there a strategy for the students to achieve that everyone should pass the exam? (*5 points*) (*Russian problem*) **B. 5067.** The midpoint of side AB of an acute angled triangle ABC is F, and the line e through the point F halves the perimeter of triangle ABC. Line e intersects the lines of sides BC and CA at points D and E, respectively. Show that the perpendiculars drawn to AB at F, to BC at D, and to CA at E are concurrent. (*5 points*) **B. 5068.** Let p be an at most 1998th-degree polynomial such that the values $p(1), p(2), \ldots, p(2000)$ form a permutation of the numbers $1, 2, \ldots, 2000$. Does