

numbers $a_1 + 1, a_2 + 2, \dots, a_n + n$ are all perfect powers? (A number is called a perfect power if it can be represented in the form a^b , where $a, b \geq 2$ are integers.) (6 points)

New problems – competition A (see page 355): **A. 755.** Prove that every polygon that has a center of symmetry can be dissected into a square such that it is divided into finitely many polygonal pieces, and all the pieces can only be translated. (In other words, the original polygon can be divided into polygons A_1, A_2, \dots, A_n , a square can be divided into polygons B_1, B_2, \dots, B_n such that for $1 \leq i \leq n$ polygon B_i is a translated copy of polygon A_i .) **A. 756.** Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ (functions with domain \mathbb{R} and values from \mathbb{R}) which satisfy the following two conditions: (i) $f(x+1) = f(x) + 1$; (ii) $f(x^2) = (f(x))^2$. (Based on a problem of *Romanian Masters of Mathematics*) **A. 757.** For every n non-negative integer let $S(n)$ denote a subset of the positive integers, for which i is an element of $S(n)$ if and only if the i -th digit (from the right) in the base two representation of n is a digit 1. Two players, A and B play the following game: first, A chooses a positive integer k , then B chooses a positive integer n for which $2^n \geq k$. Let X denote the set of integers $\{0, 1, \dots, 2^n - 1\}$, let Y denote the set of integers $\{0, 1, \dots, 2^{n+1} - 1\}$. The game consists of k rounds, and in each round player A chooses an element of set X or Y , then player B chooses an element from the other set. For $1 \leq i \leq k$ let x_i denote the element chosen from set X , let y_i denote the element chosen from set Y . Player B wins the game, if for every $1 \leq i \leq k$ and $1 \leq j \leq k$ $x_i < x_j$ if and only if $y_i < y_j$ and $S(x_i) \subset S(x_j)$ if and only if $S(y_i) \subset S(y_j)$. Which player has a winning strategy? (Proposed by *Levente Bodnár*, Cambridge)

Problems in Physics

(see page 378)

M. 388. Investigate how a piece of elastic band (either flat or round, sold in haberdasher's shops) follows Hooke's law. Measure the length of the elastic band when the applied force is increasing and also when the force is decreasing.

G. 677. We are walking at a steady rate, one step in a second. Each step is 0.5 m long. The rule is the following: one step forward, two steps backwards, then three steps forward, four steps backwards, five steps forward, six steps backwards and so on. *a)* Where are we after one minute? *b)* What is our average speed? *c)* What is our average velocity?

G. 678. A car is moving at a speed of 36 km/h, while its wheels roll without slipping. What is the velocity of the forward-most point of a wheel with respect to the ground?

G. 679. The pressure of a sample of gas in a container is measured by means of a U-shaped tube containing mercury, as shown in the *figure*. *a)* Determine the absolute pressure of the gas in the container, provided that the difference of mercury levels in the two arms of the U-shaped tube is 76 cm and the ambient air pressure is 1 atm. *b)* Then the gas is completely evacuated from the container by a pump attached to the tap shown in the figure. Determine the location of the mercury now.

G. 680. The two ends of a stretched elastic cord are jerked at the same instant, one end upward, whilst the other downward. Thus two pulses start to move towards each other as shown in the *figure*. The two symmetrical pulses carry the same amount of energy. When the two pulses meet the elastic cord becomes straight for a moment. Where is the energy of the two pulses? Will they pass each other or will they cancel each other?

P. 5143. Is it possible that the night on the Moon is so dark that only the light emitted by the stars can be seen *a)* if it is observed from the side of the Moon which faces towards the Earth, or *b)* if it is observed from the other side? **P. 5144.** A rod is fixed perpendicularly to the surface of an inclined plane of angle of elevation α . The end

of the thread of a simple pendulum of length ℓ is attached to the top of the rod. The angle between the thread of the pendulum and the surface of the slope is β . What is the period of the pendulum when it swings with small amplitude, if $\alpha + \beta < 90^\circ$, and friction is negligible? **P. 5145.** The majority of the comets, which pass the inner Solar System only once, come from the outer part of the Solar System called the Oort cloud. Estimate how long this “journey” takes for a comet. The diameter of the Oort cloud, which can be considered a spherical shell with the Sun at its centre, is 70 000 astronomical units. Suppose that the aphelion (the greatest distance measured from the Sun) of the comet is the same as the radius of the Oort cloud. **P. 5146.** The meniscus of the water in a glass is concave, whilst the meniscus of mercury in a glass container is convex at the vertical wall. Is there any shape of a glass container such that the meniscus of mercury in the container is concave at the glass wall as well? **P. 5147.** An air-plane day is organised at Pécs-Pogány airport. For advertisement purposes a helium-filled balloon of volume $V = 10 \text{ m}^3$ is launched. The balloon is tethered by a piece of rope of negligible mass. The volume of the fabric of the envelope is negligible, its mass, without the inside gas, is $m = 2 \text{ kg}$. The temperature inside the envelope, and outside, next to the balloon is $T = 300 \text{ K}$, whilst the ambient air pressure is $p_0 = 10^5 \text{ Pa}$. The pressure due to the elasticity of the fabric of the envelope can be neglected. At the end of the event the balloon is slowly winched down. The radius of the drum of the winch is $r = 10 \text{ cm}$, whilst the radius of the crank is $R = 30 \text{ cm}$. Air drag is negligible. *a)* What is the magnitude of the force applied on the crank, if the balloon is pulled down uniformly? *b)* What is the power output when the balloon is winched down, if the balloon descends at a speed of $v = 1 \text{ m/s}$? **P. 5148.** There is some air in a 1-metre long horizontal cylinder-shaped container. The container is moved horizontally parallel to its symmetry axis at a constant acceleration, whilst the temperature of the inside air is kept at a constant value of $T = 273 \text{ K}$. At what acceleration a_0 would the pressure of the air at the front of the container be *a)* 0.1% smaller than that at the back of the container; *b)* half of the value of pressure at the back of the container? *Hint:* if the temperature is constant, the density of air in the atmosphere of the Earth would change according to the barometric formula: $\rho(h) = \rho_0 e^{-\frac{Mgh}{RT}}$, where M is the average molar mass of air. **P. 5149.** A physics teacher gives a short test to two groups of students. The problem given to one of the groups is the following: “What is the angle of incidence of a thin light ray which enters into a spherical water drop and is reflected such that its path inside the drop is an equilateral triangle?” The other group is given nearly the same problem but in their case the path of the light ray in the spherical drop is a square. Can the teacher give the same problem with a regular pentagon-shaped path in the drop as a make-up test question for the missing students? Determine the angle of incidence in each case. (The refractive index of water is $\frac{4}{3}$.) **P. 5150.** We have two alike plano-convex glass lenses, with refractive index of $n = 1.5$. The plane face of one of them and the curved surface of the other is silver-coated. What is the ratio of the focal lengths of the gained two optical devices? **P. 5151.** According to the measurements, the potential values in uniform electrostatic field along a straight line segment AB as a function of the distance x measured from point A can be seen in the *table*. Determine the approximate value of the component of the electric field strength which is parallel to AB . **P. 5152.** What is the probability that a iodine-131 atom decays in the next minute? (The half-life of iodine-131 is $T_{1/2} = 8 \text{ days}$.) **P. 5153.** Two alike uniform rods are attached to each other with a pivot joint at their ends. The rods are at rest on a frictionless horizontal tabletop, lying along a straight line. Suddenly the free end of one of the rods is hit perpendicularly to the rod such that the endpoint of the rod begins to move at a speed of 1 m/s . Into what direction and at what speed does the free end of the other rod begin to move?