P. 5151. Homogén elektrosztatikus erőtérben egy AB egyenes szakaszon a mérések szerint a potenciál értéke az A-tól vett x távolság függvényében:

x [cm]	2	3	4	5	6
U [V]	130	150	180	210	230

Határozzuk meg közelítőleg a térerősség AB menti komponensének nagyságát!

(*4 pont*) Közli: *Radnai Gyula,* Budapest

P. 5152. Mennyi a valószínűsége, hogy a 131-es jód egy atomja a következő percben elbomlik? (A felezési idő: $T_{1/2} = 8$ nap.)

(*4 pont*) Közli: *Légrádi Imre,* Sopron

P. 5153. Két egyforma, homogén rúd egy-egy végpontja csuklósan kapcsolódik egymáshoz. A rudak vízszintes, súrlódásmentes asztallapon egy egyenes mentén nyugszanak. Az egyik rúd szabad végére a rúdra merőleges irányban hirtelen ráütünk, mire az a pont 1 m/s sebességgel kezd el mozogni. Milyen irányban és mekkora sebességgel indul el a másik rúd szabad végpontja?

(*6 pont*) Közli: *Gnädig Péter,* Vácduka

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Problems in Mathematics

New exercises for practice – competition K (see page 352): **K. 624.** The integers 0 to 9 are arranged along a straight line in some order. *a*) Find a possible arrangement in which the sum of any three adjacent numbers is less than 15. *b*) Is there an arrangement of this kind if 0 is not included in the numbers? **K. 625.** How many six-digit numbers are there in which each digit occurs exactly as many times as its value? **K. 626.** The table below shows the statistics of a round-robin football championship of four participating teams. Teams are listed in alphabetical order of their names. Every team played every other team exactly once. The winner of each game scored 3 points, the losing team scored 0, and in a draw each team scored 1 point.

Team	Points	Goals For	Goals Against
Headers	1	4	6
Left Foot FC		8	4
Right Foot FC		4	4
Sprinters	1	4	6

Given that the result of the Left Foot–Right Foot game was 3-1, and that the Headers scored a goal in every game they played, determine the result of each individual game.

K. 627. The teacher selects a student from a class at random. The probability of selecting a boy is 2/3 of the probability that he selects a girl. What fraction of the whole class are girls? **K. 628.** Four identical rectangular sheets of paper are placed on the table to form a larger rectangle without gaps or overlaps. The area of the resulting rectangle is $1200\ \mathrm{cm}^2$. Given that it is not possible to transfer any rectangle to any other only by translation, find the perimeter of the large rectangle.

New exercises for practice – competition C (see page 353): **Exercises up to grade 10: C. 1553.** Determine the constant term of the expression $\left(x^{12}+\frac{1}{x^{18}}\right)^{25}$. **C. 1554.** One side of a rectangle is $\frac{1+\sqrt{5}}{2}$ times as long as the other side. The rectangle is dissected and put together to form a square of the same area. What is the ratio of the diagonal of the rectangle to the diagonal of the square? **Exercises for everyone: C. 1555.** Solve the equation $x+y^2=4z^2$ over the set of positive prime numbers. **C. 1556.** The interior angle bisector drawn from vertex C of triangle ABC intersects the opposite side at point P. The distance of point P from the sides is $\frac{24}{11}$, and $AC=6$, $BC=5$. Find the length of side AB. **C. 1557.** Two numbers are selected at random from the set of two-digit positive integers. What is the probability that they will have a digit in common? **Exercises upwards of grade 11: C. 1558.** Depending on the value of the nonzero parameter a, how many points do the circle $x^2+y^2=1$ and the parabola $y=ax^2-1$ have in common? **C. 1559.** The base of a tetrahedron is a regular triangle, and the three lateral faces unfolded and laid on the plane form a trapezium with sides 10, 10, 10 and 14 units of length. Find the sum of the lengths of all edges of the tetrahedron, and also find its surface area.

New exercises – competition B (see page 354): **B. 5038.** Let P be a point in the interior of a regular octagon $ABCDEFGH$. Show that the sum of the areas of triangles ABP, CDP, EFP and GHP equals the sum of the areas of triangles BCP, DEP, FGP and HAP. (*3 points*) **B. 5039.** Every entry in a 2019×2019 table is either $(+1)$ or (-1). If the sum of each row and the sum of each column are calculated, how many different numbers may be obtained at most? (*3 points*) (Proposed by *I. Blahota,* Nyíregyháza) **B. 5040.** In a square $ABCD$, let F be an interior point of side AB, and let E be an interior point of side AD. Draw a perpendicular to line CE at point E, and a perpendicular to line CF at point F. Denote the intersection of the two perpendiculars by M. Given that the area of triangle CEF is half the area of pentagon $BCDEF$, prove that point M lies on diagonal AC of the square. (*4 points*) **B. 5041.** An $n\times n$ table of real numbers in each field is called a zero square if the sum of the numbers in every 2×2 square part of it (therefore in the whole table, too) is zero. (The *diagram* shows a 3×3 example.) What is the largest possible n for which there exists an $n\times n$ zero square such that the entries are not all zeros? (*5 points*) **B. 5042.** The convex quadrilateral $ABCD$ is not a trapezium, and diagonals AC and BD are equal in length. Let M denote the intersection of the diagonals. Show that the other intersection (different from M) of the circles ABM and CDM lies on the angle bisector of angle BMC. (*4 points*) **B. 5043.** Prove that the set $\{1,2,3,4,5,6,7,8,9,10,11,12,13\}$ has an odd number of nonempty subsets in which the arithmetic mean of the elements is an integer. (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5044.** Let D be a point in the interior of side AB in triangle ABC, and E be a point in the interior of side AC. The intersection of line segments BE and CD is M. Let x denote the area of triangle BCM, and let y denote the area of triangle EDM. Prove that $T_{ABC}\geqslant x\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$. (*6 points*) **B. 5045.** For which positive integers n is there an appropriate order $a_1,a_2,\ldots,a_n$ of the first n positive integers such that the numbers $a_1+1, a_2+2, \dots, a_n+n$ are all perfect powers? (A number is called a perfect power if it can be represented in the form a^b, where $a, b \geqslant 2$ are integers.) (*6 points*)

New problems – competition A (see page 355): **A. 755.** Prove that every polygon that has a center of symmetry can be dissected into a square such that it is divided into finitely many polygonal pieces, and all the pieces can only be translated. (In other words, the original polygon can be divided into polygons $A_1, A_2, \dots, A_n$, a square can be divided into polygons a $B_1, B_2, \dots, B_n$ such that for $1 \leqslant i \leqslant n$ polygon B_i is a translated copy of polygon A_i.) **A. 756.** Find all functions $f: \mathbb{R} \to \mathbb{R}$ (functions with domain $\mathbb{R}$ and values from $\mathbb{R}$) which satisfy the following two conditions: (*i*) $f(x+1) = f(x)+1$; (*ii*) $f(x^2) = \big(f(x)\big)^2$. (Based on a problem of *Romanian Masters of Mathematics*) **A. 757.** For every n non-negative integer let $S(n)$ denote a subset of the positive integers, for which i is an element of $S(n)$ if and only if the i-th digit (from the right) in the base two representation of n is a digit 1. Two players, A and B play the following game: first, A chooses a positive integer k, then B chooses a positive integer n for which $2^n \geqslant k$. Let X denote the set of integers $\{0, 1, \dots, 2^n-1\}$, let Y denote the set of integers $\{0, 1, \dots, 2^{n+1}-1\}$. The game consists of k rounds, and in each round player A chooses an element of set X or Y, then player B chooses an element from the other set. For $1 \leqslant i \leqslant k$ let x_i denote the element chosen from set X, let y_i denote the element chosen from set Y. Player B wins the game, if for every $1 \leqslant i \leqslant k$ and $1 \leqslant j \leqslant k$ $x_i < x_j$ if and only if $y_i < y_j$ and $S(x_i) \subset S(x_j)$ if and only if $S(y_i) \subset S(y_j)$. Which player has a winning strategy? (Proposed by *Levente Bodnár,* Cambridge)

Problems in Physics

(see page 378)

M. 388. Investigate how a piece of elastic band (either flat or round, sold in haberdasher's shops) follows Hooke's law. Measure the length of the elastic band when the applied force is increasing and also when the force is decreasing.

G. 677. We are walking at a steady rate, one step in a second. Each step is 0.5 m long. The rule is the following: one step forward, two steps backwards, then three steps forward, four steps backwards, five steps forward, six steps backwards and so on. *a*) Where are we after one minute? *b*) What is our average speed? *c*) What is our average velocity? **G. 678.** A car is moving at a speed of 36 km/h, while its wheels roll without slipping. What is the velocity of the forward-most point of a wheel with respect to the ground? **G. 679.** The pressure of a sample of gas in a container is measured by means of a U-shaped tube containing mercury, as shown in the *figure. a*) Determine the absolute pressure of the gas in the container, provided that the difference of mercury levels in the two arms of the U-shaped tube is 76 cm and the ambient air pressure is 1 atm. *b*) Then the gas is completely evacuated from the container by a pump attached to the tap shown in the figure. Determine the location of the mercury now. **G. 680.** The two ends of a stretched elastic cord are jerked at the same instant, one end upward, whilst the other downward. Thus two pulses start to move towards each other as shown in the *figure.* The two symmetrical pulses carry the same amount of energy. When the two pulses meet the elastic cord becomes straight for a moment. Where is the energy of the two pulses? Will they pass each other or will they cancel each other?

P. 5143. Is it possible that the night on the Moon is so dark that only the light emitted by the stars can be seen *a*) if it is observed from the side of the Moon which faces towards the Earth, or *b*) if it is observed from the other side? **P. 5144.** A rod is fixed perpendicularly to the surface of an inclined plane of angle of elevation α. The end