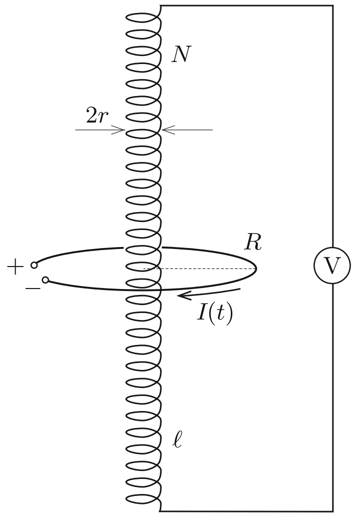
P. 5129. Egy r sugarú, N menetszámú, igen hosszú, $n=N/\ell$ menetsűrűségű szolenoidot az *ábrán* látható módon egy $R \ll \ell$ sugarú körvezetővel vettünk körül. Mekkora értéket mutat a szolenoid végpontjai közé kapcsolt ideális voltmérő, ha a körvezetőbe időben egyenletesen, $I(t)=\alpha\cdot t$ módon változó áramot vezetünk?

(*5 pont*) Közli: *Vigh Máté,* Budapest



P. 5130. Hány fényév távolságra van tőlünk az a galaxis, amelynek egyik csillagáról hozzánk érkező sugárzásban a hidrogén 4d → 2p átmenetnek megfelelő fény hullámhossza 513 nm?

(*5 pont*) Közli: *Zsigri Ferenc,* Budapest

P. 5131. Három azonos, állandó hőkapacitású test közül kettőnek a hőmérséklete 300 K, a harmadiké 100 K. Fel lehet-e melegíteni valamelyik testet 400 K hőmérsékletre külső hő és munka befektetése nélkül, csupán termodinamikai gépeket (hőerőgép, hűtőgép) működtetve a testek között?

(*6 pont*) Közli: *Radnai Gyula,* Budapest

Áprilisi pótfeladat.* Tréfásan fogalmazva: az akcióhős Chuck Norris fekvőtámasz végzésekor nem is a saját testét emeli meg, hanem valósággal eltolja a Földet. Mennyi az igazság ebben?

Közli: *Vass Miklós,* Budapest

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Problems in Mathematics

New exercises for practice – competition C (see page 226): **Exercises up to grade 10: C. 1539.** Let E denote the point on side AB of a square $ABCD$ which divides the side 1 : 3, with the shorter segment lying closer to A. Let F be an arbitrary

[1] A megoldás beküldhető, de nem számít bele a pontversenybe.

point of diagonal BD. Determine the minimum of the sum $AF+EF$. **C. 1540.** The coefficients of the quadratic expression ax^2+bx+c are integers, and $a>0$. It has two distinct positive roots smaller than 1. Find the smallest possible value of a. **Exercises for everyone: C. 1541.** Prove that there exists a sequence of 2019 consecutive positive integers that includes exactly 19 primes. **C. 1542.** The lengths of the legs in a right-angled triangle ABC are 5 and 12. Let P, Q and R be points on the inscribed circle of the triangle such that triangle PQR is similar to triangle ABC. Determine the lengths of the sides of triangle PQR. **C. 1543.** For what values of the positive integer n will 2^n+1 or 2^n-1 be divisible by 9? **Exercises upwards of grade 11: C. 1544.** The diagonals of a circumscribed trapezium $ABCD$ intersect at E. The radii of the inscribed circles of triangles ABE, BCE, CDE and DAE are r_1, r_2, r_3 and r_4, respectively. Prove that $\frac{1}{r_1}+\frac{1}{r_3}=\frac{1}{r_2}+\frac{1}{r_4}$. **C. 1545.** Find the real solutions of $x^2-y^2=\log_2\frac{y}{x}$, $3^{x^2+y^2-1}-4\cdot 3^{xy}+9=0$. (*Romanian competition problem*)

New exercises – competition B (see page 227): **B. 5022.** Given some unit circles on the plane, we coloured each centre blue. On the circumferences of the circles, we marked some points red such that there should be exactly 2 red points on the circumference of each circle. What is the maximum possible number of blue points if there are 25 coloured points altogether? (*3 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5023.** In a triangle ABC, $\angle ACB=90^\circ$ and $AC>BC$. Let X be the midpoint of the arc AB of the circumscribed circle that does not contain C. The perpendicular drawn to CX at X intersects line CA at P. Show that $AP=BC$. (*3 points*) (Proposed by *L. Surányi,* Budapest) **B. 5024.** Let p denote an odd prime. If each of the numbers $\binom{p-2}{0},\binom{p-2}{1},\dots,\binom{p-2}{p-2}$ are divided by p, how many different remainders are obtained? (*4 points*) (Proposed by *Z. Gyenes* and *B. Hujter,* Budapest) **B. 5025.** The inscribed circle of triangle ABC is centred at I, and touches sides BC, CA and AB at points D, E and F, respectively. Let M be an arbitrary point in the interior of side BC, different from D. Let the lines DI and EF intersect at T, and let K denote the midpoint of line segment MT. Prove that the circles DEF, TDM and KIT are concurrent. (*5 points*) (Proposed by *M. Agazade,* Azerbaijan) **B. 5026.** Let P be an arbitrary point of a given ellipse, different from the endpoints of the major axis. P is connected to the foci F_1 and F_2. The angle bisector of angle $\angle F_1PF_2$ intersects F_1F_2 at E. The circle which passes through P and touches F_1F_2 at E intersects PF_1 at G and PF_2 at H. Show that the length of GH does not depend on the choice of P. (*4 points*) (Proposed by *L. Németh,* Fonyód) **B. 5027.** Arthur Dumpling (Hungarian cartoon character: a fat bird who loves chocolate of all kinds) lives at 1 Sweet Street. The chocolate shop is operating at number n, the far end of the street. Arthur's daily fitness programme is as follows: he starts in front of number 2. When he stands in front of number k (where $1<k<n$), he tosses a fair chocolate coin. If it shows heads, he moves to number $(k-1)$. If it shows tails, he moves to number $(k+1)$. If he reaches the chocolate shop, he enters and throws a chocolate ball down his throat, and then moves to number $(n-1)$. If he arrives back home, the fitness programme terminates. On average, how many chocolate balls does Arthur throw down his throat per day? (*5 points*) **B. 5028.** Let us define a function f as follows. For any acute-angled triangle XYZ, if P is a point on YZ, then $f(P;XYZ)$ is defined as the line joining the feet of perpendiculars from P to lines XY; XZ. Let ABC be a triangle with orthocenter H. Let $A'B'C'$ be the orthic triangle of ABC. Let $A''\equiv f(B';HCA)\cap f(C';HAB)$. Similarly, points B''; C'' are defined. Show that the lines AA''; BB''; CC'' are concurrent. (*6 points*) (Proposed by *K. V. Sudharshan*) **B. 5029.** Assume that a certain football team have played 1000 games

altogether, and scored 1000 points altogether since the team was founded. (A team score 3 points for every game they win, 1 point for a draw and no points for games they lose.) Prove that there are at most $(2.9)^{1000}$ possible sequences of the 1000 scores. (*6 points*)

New problems – competition A (see page 228): **A. 749.** Given are two polyominos, the first one is an L-shape consisting of three squares, the other one contains at least two squares. Prove that if n and m are co-prime then at most one of the $n\times n$ and $m\times m$ boards can be tiled by translated copies of the two polyominos. (Proposed by: *András Imolay, Dávid Matolcsi, Ádám Schweitzer* and *Kristóf Szabó,* Budapest) **A. 750.** Let $k_1,\dots,k_5$ be five circles in the plane such that k_1 and k_2 are externally tangent to each other at point T, k_3 and k_4 are externally tangent to both k_1 and k_2, k_5 is externally tangent to k_3 and k_4 at points U and V, respectively, moreover k_5 intersects k_1 at P and Q, like shown in the *figure.* Show that $\frac{PU\cdot PV}{QU\cdot QV}=\frac{PT^2}{QT^2}$. **A. 751.** Let $c>0$ be a real number, and suppose that for every positive integer n, at least one percent of the numbers $1^c,2^c,3^c,\dots,n^c$ are integers. Prove that c is an integer.

Problems in Physics

(see page 249)

M. 386. Make a long-period torsion pendulum, which swings in air. With measurement determine how the period of the pendulum depends on the length of the thread.

G. 669. Signs similar to the one shown in the figure can often be seen on highways to warn drivers to maintain a safe following distance. How can the following distance be given in seconds? Why is it that the appropriate "following distance" is two or more seconds? **G. 670.** Different amount of samples of water of different temperature values are mixed. The corresponding volume and temperature data are the following: 1 litre water at 10 °C, 2 litres water at 20 °C, 3 litres water at 30 °C, 4 litres water at 40 °C, 5 litres water at 50 °C, 6 litres water at 60 °C, 7 litres water at 70 °C, 8 litres water at 80 °C and 9 litres water at 90 °C. What is the common temperature of the mixture if heat losses are negligible? **G. 671.** Two large, vertical and parallel plane mirrors are facing opposite to each other at a distance of 1 m. If you stand exactly midway between the mirrors outstretching your hand sideways, and observe the image of your palm reflected in one of the mirrors, you see quite a lot of images. What are the distances between the images of your *palm*? **G. 672.** A room has three doors, and next to each door there is a switch. With each of them the chandelier in the room can be switched on or off separately. In order to build such a circuit, multiway switches, namely 3-way and 4-way switches should be used. How can this circuit be built? Look up in literature how these switches work, and give the diagram of their connections.

P. 5122. The braking distance of a car moving along dry and horizontal, asphalt-covered road at a speed of 50 km/h is at least 13 m, that is, the distance the car covers from the instant when the brakes are applied to when it comes to a complete stop. (In the definition of the braking distance the reaction time of neither the driver nor the vehicle are included.) What is the minimum braking distance of the same car at a speed of 20 km/h on an unusually steep slope of angle of elevation of 30° (approximately 58% slope)? Investigate both the upward and downward motions. **P. 5123.** There is a small object of mass $m=0.25$ kg at the left end of a trolley of mass $M=1$ kg and of length $L=0.3$ m. The trolley has vertical walls at its sides, and moves frictionlessly along the horizontal plane. Its small wheels are of negligible mass and size. At a given instant the small object