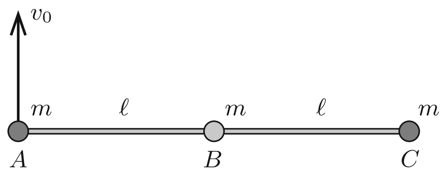
P. 5121. Három (A, B és C jelű) kicsiny, egyforma, m tömegű golyó úgy van összekötve két elhanyagolható tömegű, ℓ hosszúságú rúddal, hogy az egyik rúd az A és a B golyót, a másik rúd a B és a C golyót köti össze. A B golyónál a kapcsolódás csuklós, így a rudak közötti szög akadálytalanul változhat. A rendszer a súlytalanság állapotában nyugalomban van, és a három golyó egy egyenes mentén helyezkedik el. Ekkor az A golyónak pillanatszerűen a rudakra merőleges, v_0 nagyságú sebességet adunk. Mekkora erő hat a rudakban az indítást követő pillanatban?



(*6 pont*) *Olimpiai versenyfeladat nyomán*

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Problems in Mathematics

New exercises for practice – competition K (see page 159): **K. 619.** What is the largest possible number of primes such that the sum of any three of them is also a prime? **K. 620.** The sum of five positive integers is 20. The absolute values of their pairwise differences are 1, 2, 3, 3, 4, 5, 6, 7, 9, 10. Find all such sets of five numbers. **K. 621.** Nine members of a math club are designing a 3×3 square flag as shown in the *figure*. In the nine fields, they arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 so that the sum of the numbers in each row, each column, and each diagonal is divisible by 3. How many different flags may they make? **K. 622.** The 16 tokens in the game of QUARTO are all different from each other in some property. The tokens can be categorized into two sets of the same number of elements in four different ways: – tall or flat; – black or white; – round or square; – with or without a hole on the top. Is it possible to arrange the 16 tokens in a circle so that adjacent ones should have exactly two properties in common? **K. 623.** The front side of a square sheet of paper $ABCD$ is red, and the back side is white. E and F divide diagonal AC into three equal parts, with E lying closer to A. The sheet is folded along lines perpendicular to AC by folding the back side towards the front (that is, making the back of the sheet appear on top). During the first folding, point A is moved to cover F, and during the second folding, point C is moved to cover E. What will be the ratio of the red area to the white area on the front side of the sheet in the end?

New exercises for practice – competition C (see page 160): **Exercises up to grade 10: C. 1532.** Show that if a, b, c are positive numbers and $a+b+c\geqslant \frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}$, then one of them is at least 1. **C. 1533.** The perimeter of a right-angled triangle is k, one of the legs is b, and the opposite angle is β. Consider the triangle in which there are two sides of lengths k and $b\cdot\sqrt{2}$, and they enclose an angle of 45°. Find the smallest angle of this triangle. **Exercises for everyone: C. 1534.** Find all real pairs (x,y) satisfying $5x^2+y^2-4xy+24\leqslant 10x-1$. **C. 1535.** Prove that if the area of a convex quadrilateral is halved by each diagonal, then the quadrilateral is a parallelogram. **C. 1536.** Find all real pairs (x, y) satisfying $xy = x + y + 5$, $x^2 + y^2 = 5$. **Exercises upwards of grade 11: C. 1537.** A circle k_1 of radius 6 and a circle k_2 of radius 3 touch each other on the outside, and each of them touches a circle k of radius 9 on the inside. One common exterior tangent of k_1 and k_2 intersects circle k at points P and Q. Determine the length of the line segment PQ. (*Croatian problem*) **C. 1538.** Six pairs of twin brothers participated in one of the practices of the Twins' Table Tennis Club. The coaches did not want any brothers to play at the same table. *a*) In how many different ways may they divide the players to play round-the-table games at two different tables? *b*) In how many different ways is it possible to divide the players into sets of four to play doubles at three different tables? (The position of the players at the tables does not matter.) (*Based on an English problem*)

New exercises – competition B (see page 162): **B. 5014.** After the elections in Nowhereland, there are $50 < n < 100$ representatives in the parliament, all from a single party called the Blue Party. (The Blue Party has a single president.) According to the law, a party in the parliament may be divided into two parties as long as the following conditions are met: • The president of the old party is not allowed to become a member of the newly formed parties. His or her parliament mandate will terminate, thereby reducing the total number of representatives. • Every other member may decide which new party to join. • Each of the new parties must have at least one member among the representatives. • Each of the new parties must elect a president from their representatives. If at least one such splitting of a party results in all parties in the parliament having the same number of members, the parliament will be dissolved. What should be the value of n so that this could never happen? (*3 points*) **B. 5015.** The second intersections of three concurrent unit circles are A, B and C. What is the radius of the circle ABC? (*3 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5016.** In a convex quadrilateral $ABCD$, point E_1 lies on side AD, point F_1 lies on side BC, E_2 lies on diagonal AC, and F_2 lies on diagonal BD. Given that $AE_1 : E_1D = BF_1 : F_1C = AE_2 : E_2C = BF_2 : F_2D = AB : CD$ and no pair of points coincide, prove that the lines E_1F_1 and E_2F_2 are perpendicular. (*4 points*) **B. 5017.** Is there a function $f : \mathbb{R} \to \mathbb{R}$ with the following properties: (1) if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$, (2) there exist appropriate constants $a, b > 0$ such that $f(x^2) - \big(f(ax + b)\big)^2 \geqslant \frac{1}{4}$ for all $x \in \mathbb{R}$? (*4 points*) **B. 5018.** The sultan imprisoned all the 1024 mathematicians of his empire. They were not allowed to keep any of their possessions except for a single copper coin each. The mathematicians know that there are 1024 of them, but they are not able to communicate with one another in any way. On his birthday the sultan offered them the following game: they are taken to the prison yard one by one. Each of them may say either 0 or 1 when taken there. If the sum of the numbers they say is 1, then he will let them all go free. (The mathematicians cannot signal to each other, they do not know how many others have been to the yard before them, or what those before them have done in the yard.) What are the chances that they can get out of the prison? (*5 points*) **B. 5019.** The quadrilateral $ABCD$ is cyclic. Given that $AB + BC = AD + DC$ and $BA + AC = BD + DC$, show that $ABCD$ is a rectangle. (*6 points*) **B. 5020.** A parabola is reflected in a line that passes through its focus and encloses an angle α with its axis. Show that the parabola and its reflection intersect at an angle of α. (*5 points*) (Proposed by *L. Németh,* Fonyód) **B. 5021.** A positive integer n is not divisible by 3. The sum of its positive divisors that leave a remainder of 1 when divided by 3 is $A(n)$, and the sum of its positive divisors that leave a remainder of 2 when divided by 3 is $B(n)$. Find those numbers n for which $\big|A(n) - B(n)\big| < \sqrt{n}$. (*6 points*)

New problems – competition A (see page 163): **A. 746.** Let p be a prime number. How many solutions does the congruence $x^2 + y^2 + z^2 + 1 \equiv 0 \pmod{p}$ have among the modulo p remainder classes? (Proposed by: *Zoltán Gyenes,* Budapest) **A. 747.** In a simple graph on n vertices, every set of k vertices has an odd number of common neighbours. Prove that $n+k$ must be odd. (Proposed by: *András Imolay, Dávid Matolcsi, Ádám Schweitzer* and *Kristóf Szabó,* Budapest) **A. 748.** The circles Ω and ω in its interior are fixed. The distinct points A, B, C, D, E move on Ω in such a way that the line segments AB, BC, CD and DE are tangents to ω. The lines AB and CD meet at point P, the lines BC and DE meet at Q. Let R be the second intersection of the circles BCP and CDQ, other than C. Show that R moves either on a circle or on a line. (Proposed by: *Carlos Yuzo Shine,* Sao Paolo)

Problems in Physics

(see page 185)

M. 385. When a relatively large obstacle having a horizontal (planar) face is placed under the water stream flowing from the kitchen tap, then the spreading water forms a circular area where it has a visible increase in its height. This is called the hydraulic jump. At a certain obstacle-tap distance, measure how the radius of the circle depends on the rate of water flow.

G. 665. In this problem the motion of a small disc sliding along a horizontal frictionless surface of ice is investigated. A column, having a square-shaped cross section, emerges from the ice. The side of the square is 10 cm. The disc is attached to the column by means of a piece of 1.0 m long thread. The top view of the arrangement is shown in the *figure.* The disc is given an initial speed of $v=1.0$ m/s. How much time elapses until the disc hits the column? **G. 666.** The *figure* shows the sketch of the structure of an amusement park ride. The big cylinder-shaped pole at the centre is rotating uniformly. By four horizontal struts circular "gondolas" are attached to the pole and move around it. At the centre of each gondola a horizontal disc is fixed to the gondola such that the symmetry axis of the disc coincides with the vertical shaft of the gondola about which it can be rotated. These discs on the gondolas are connected by transmission belts to disc K, which is attached to the pole at the centre of the structure. Disc K is fixed, and not rotating at all. (For clarity reasons only one of the transmission belts was drawn in the figure.) At points A, B, C and D there is a passenger in each gondola. What is the path of each passenger? How does the distance between them change during the rotation? (The structure is rotating in the horizontal plane and all the rotational axes are vertical.) **G. 667.** There is an aluminium cube of edge 10 cm on a table. What is the pressure due to the cube on the table? By what percent does this pressure change when the temperature of the cube is increased from 20 °C to 100 °C? Does it increase or decrease? **G. 668.** Watch the following YouTube video: `https://www.youtube.com/watch?v=hvqQ1XG1aQE`, make your own button spinner (buzzer) from a button of appropriate size and a piece of thin thread, and then try it. Why does the button begin to spin fast?

P. 5111. A ping-pong ball was thrown vertically upward. Which takes longer, the upward or the downward motion of the ball? (Consider air drag.) **P. 5112.** From a wall of height H a snowball was thrown at an initial speed of v_0 and at an angle of α with respect to the horizontal. A child, who was at a distance of s from the wall, began to run at the same moment when the snowball was thrown. What was the initial direction and speed of the child, if he ran at a constant speed along a straight line and the snowball hit him? (Air drag is negligible.) Both the motions of the child and the snowball are in a vertical plane, which is perpendicular to the wall. *Data:* $H=45$ m, $s=21$ m, $v_0=5$ m/s, $\alpha=30°$. **P. 5113.** An 80-kg man is walking up in a tower built on the Equator of the Earth. How much does the apparent weight of the man decrease in each metre of ascent? **P. 5114.** Ending at the rim of the table there is a slope of angle of elevation of α, from which a uniform density rectangular block of length ℓ and of height d is sliding down. By what length does the block move further from the end of the table until it tilts, if a) friction between the slope and the block is negligible; b) the coefficient of kinetic