P. 5109. Mekkora az elektron hullámhossza, ha a mozgási energiája

a) $1{,}75\cdot 10^{-16}$ J;

b) 20 GeV?

(*5 pont*) Közli: *Légrádi Imre,* Sopron

P. 5110. A Föld körül keringő két mesterséges hold pályájának fél nagytengelye ugyanakkora. A holdak pálya menti sebességeinek aránya a perigeumban (földközelpontban) $\frac{3}{2}$, és az itt nagyobb sebességű hold pályájának excentricitása 0,5.

Határozzuk meg pálya menti sebességük arányát az apogeumban (földtávolpontban), és számítsuk ki a másik mesterséges hold pályájának excentricitását!

(*6 pont*) *Csillagászati versenyfeladat nyomán*

Beküldési határidő: 2019. március 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

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Problems in Mathematics

New exercises for practice – competition K (see page 94): **K. 614.** In the increasing sequence of positive integers (starting with 1), find the 225th number that cannot be represented as a product of two consecutive integers. **K. 615.** Six points are selected in the interior of a square such that no three points among the 10 points (the vertices of the square and the six points) are collinear. From these 10 points, pairs of points are connected with line segments that do not intersect each other, and the process is continued until it is not possible to add a further line segment. What is the largest possible number of line segments that may be drawn in this way? **K. 616.** There are a lot of integers that can be represented as a sum of three perfect squares. For example, $1=1^2+0^2+0^2$, $14=3^2+2^2+1^2$, $20=4^2+2^2+0^2$. Show that 1991 cannot be represented as a sum of three perfect squares. **K. 617.** The diagonal AG of a rectangular block $ABCDEFGH$ intersects the triangle BDE at point Q. Prove that Q is the centroid of triangle BDE. **K. 618.** A positive integer n is said to be a "strong" number if its number of divisors is greater than the number of divisors of each positive integer less than n. (For example, $n=2$ is a strong number, because it has two divisors, while $n=1$ has only one. But $n=3$ is not a strong number, because it has two divisors similarly to a smaller integer $n=2$.) *a*) Find all strong numbers greater than 2 but less than 30. *b*) Is $2^3\cdot 3^4\cdot 5$ a strong number?

New exercises for practice – competition C (see page 95): **Exercises up to grade 10: C. 1525.** A team in a football championship had 33 points after 15 games they played. The 15 games included all three kinds of outcome: winning, losing, and draw. How many games did they win? (3 points are scored for winning a game, 0 for losing and 1 for each team in the case of a draw.) **C. 1526.** The circumscribed circle of a square is reflected in each side. Let T denote the area of the circle that touches these reflections in the interior of the square. Let t denote the area of the circle that touches one reflection and the circumscribed circle, both from the inside. Determine the smallest possible value of $\frac{T}{t}$. **Exercises for everyone: C. 1527.** If two appropriate numbers in the sequence $1, 2, \ldots, n$ are erased, the sum of the remaining numbers will be 2019. Find all possible pairs of numbers that may be erased. **C. 1528.** What positive integer may n denote if the number obtained by erasing the last three digits of the number n^3 is n itself? (Proposed by *S. Róka,* Nyíregyháza) **C. 1529.** Prove that every right-angled triangle can be divided into $3k+2$ isosceles triangles where k is any positive integer. **Exercises upwards of grade 11: C. 1530.** Is it possible to group all the integers from 1 to 51 into sets of three such that the sum of the numbers in each set should be a prime? **C. 1531.** The base of a right prism is a regular triangle, and its volume is 2 dm^3. What is the minimum possible surface area of the prism?

New exercises – competition B (see page 96): **B. 5006.** The bases of a trapezium $ABCD$ are AB and CD, the intersection of the diagonals is M. Diagonal AC bisects the angle BAD, $AM=BC$ and $BM=CD$. Find the angles of the trapezium. (*4 points*) (Based on a problem of the National Competition) **B. 5007.** We have $3n+1$ coins, n of which have 0 on one side and 11 on the other. Another n coins have 0 on one side and 44 on the other, and the remaining $n+1$ coins have 0 on one side and 99 on the other. All the coins are tossed at the same time. What is the probability that the sum of the resulting numbers is divisible by 7? (The coins are fair coins.) (*4 points*) **B. 5008.** A circle k_A is centred at A, and a circle k_B is centred at B. Line l_1 touches k_A at A_1 and k_B at B_1. Line l_2 touches k_A at A_2 and k_B at B_2. Prove that the orthogonal projections of the line segments A_1A_2 and B_1B_2 onto the line AB are of equal length. (*3 points*) **B. 5009.** Given that $x^2+y^2+z^2=3$, where x, y, z are positive numbers, prove that $2^{\frac{1}{x}}+2^{\frac{1}{y}}+2^{\frac{1}{z}}\geqslant 6$. (*3 points*) (Proposed by *V. N. Nguyen,* Vietnam) **B. 5010.** The inscribed circle of an acute-angled triangle ABC touches the sides at the points A_0, B_0 and C_0. The points of tangency of the excircles on the lines of the sides are A_1, B_1 and C_1; A_2, B_2 and C_2; A_3, B_3 and C_3, respectively. Let T_i denote the area of triangle $A_iB_iC_i$ $(i=0,1,2,3)$. Show that $\frac{1}{T_0}=\frac{1}{T_1}+\frac{1}{T_2}+\frac{1}{T_3}$. (*5 points*) **B. 5011.** We are given 6 points in the plane such that all pairwise distances are different. Show that there exist two triangles with the following property: each vertex is among the 6 given points, and the two triangles share a side in common such that this side is the shortest one in one triangle and the longest one in the other. (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5012.** Let $f(x)$ be a polynomial with integer coefficients. Let $f^{(n)}$ denote the n-fold application of f: $f^{(n)}(x)=\underbrace{f\Big(f\big(\ldots f}_{n}(x)\ldots\big)\Big)$. Let $k(f)$ denote the smallest positive integer k for which $f^{(k)}(x)\equiv x \pmod{13}$ holds for all integers x, provided that there exists such a k, and let $k(f)=0$ otherwise. Prove that there is a maximum value of $k(f)$, and determine this maximum value. (*6 points*) **B. 5013.** The excircle of triangle ABC opposite to vertex A touches line AC at point B_1. The line segment BB_1 intersects the excircle at B_2, and the tangent drawn to the excircle at B_2 intersects side BC at B_3. Similarly, the inscribed circle of the triangle touches side AB at point C_1, line segment CC_1 intersects the incircle at C_2, and the tangent drawn to the incircle at C_2 intersects side BC at C_3. Prove that $B_2B_3=C_2C_3$. (*6 points*)

New problems – competition A (see page 97): **A. 743.** The incircle of tangential quadrilateral $ABCD$ intersects diagonal BD at P and Q $(BP<BQ)$. Let UV be the diameter of the incircle perpendicular to AC $(BU<BV)$. Show that the lines AC, PV and QU pass through one point. (Based on problem 2 of *IOM 2018,* Moscow) **A. 744.**

Show that for every odd integer $N > 5$ there exist vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in (three-dimensional) space which are pairwise perpendicular, not parallel with any of the coordinate axes, have integer coordinates, and satisfy $|\mathbf{u}| = |\mathbf{v}| = |\mathbf{w}| = N$. (Based on problem 2 of the *2018 Kürschák contest*) **A. 745.** We have attached a clock hand to every face of a convex polyhedron. Each hand always points towards a neighboring face (two faces of the polyhedron are neighbors if they share an edge). At the end of every minute, exactly one of the hands turns clockwise to point at the next face. Suppose that the hands on neighboring faces never point towards one another. Show that one of the hands makes only finitely many turns.

Problems in Physics

(see page 122)

M. 384. Determine the efficiency of an electric kettle of known power rating, as a function of the mass of the water in it.

G. 661. Three types of liquid, which do not mix with each other, are poured into a graduated cylinder: 100 g water of density 1000 kg/m^3, 200 g oil of density 0.8 g/cm^3, and mercury, such that the graduated cylinder of volume 400 cm^3 and of height 40 cm is fully filled. How many grams of mercury was poured into the cylinder? At what height, measured from the bottom of the cylinder, are the boundary layers which separate the different liquids? (The density of mercury is 13 600 kg/m^3.) **G. 662.** A disc with greater radius and two smaller cylinder are attached to each other concentrically; the assembly is shown in *figures* (*a*) and (*b*). Two pieces of cords were wrapped around the cylinders, and their ends are moved horizontally at constant speed of v, with the help of a rod. In the (*c*) case, there is another cylinder, which can rotate freely, above the cylinder on the original device. The cylinders have the same radius, and touch each other tightly. A rod structure is attached to the cylinders in order to prevent them from falling. A piece of cord is wrapped around the top cylinder and the end is again pulled at a constant speed of v. The discs do not slip on the ground and the cylinders do not slip on each other. In what direction will the centre of the disc move in each case? Will the speed of the centre of the disc be greater or smaller than v in each case? **G. 663.** There are two filament lamps, a battery and a double switch in the *figure.* The switch changes two contacts at the same time when it is turned. Plan a circuit (that is, draw the wires) using the given components in which the two lamps are connected in series at one position of the switch, and when the switch is turned, then the lamps are connected in parallel. **G. 664.** In which case are we closer to the Sun on the same day of the year: at new-moon or at full-moon? Estimate the difference between the distances at the two cases?

P. 5100. Two cannons fire a ball at the same time towards each other from points A and B as shown in the *figure.* The nozzle speed of the ball fired from the cannon at A is 40 m/s, whilst the nozzle speed of the ball fired from the cannon at B is 60 m/s. Do the two balls collide? If yes, where and when? If not, where do the balls hit the ground? **P. 5101.** A space ship orbits along a circular path around the Earth, its period is 100 minutes. What is the area of the surface of the Earth which is seen by the astronaut at a certain instant? (Neglect the refraction of light due to the atmosphere.) **P. 5102.** A trolley of mass m_1 is moving at a speed of v_0 along the horizontal floor towards another trolley of mass m_2, which is at rest. On the top of each trolley there is a thin rectangular block of mass m. The coefficient of static friction between the blocks and the surface of the trolleys is μ_0. There is a spring of spring constant D on the stationary trolley. Will any of the blocks slide due to the collision? *Data:* $m_1 = 0.2$ kg; $m_2 = m = 0.1$ kg; $\mu_0 = 0.5$; $D = 12$ N/m; $v_0 = 1$ m/s. **P. 5103.** A slinky of mass m is suspended at one of its ends, due to its own