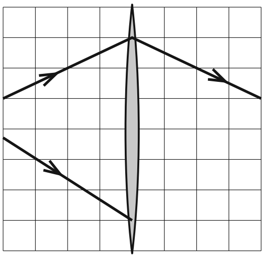
P. 5085. A mellékelt (méretarányos) *ábra* felső felén egy vékony, hagyományos gyűjtőlencsén áthaladó fénysugár menete látható. Hogyan fog továbbhaladni ugyanezen a lencsén az *ábra* alsó felén látható fénysugár?



(*4 pont*) Közli: *Vigh Máté,* Budapest

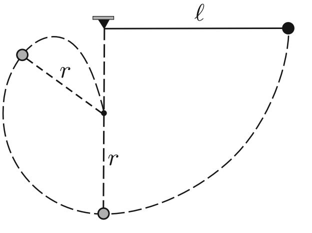
P. 5086. Mekkora energia szükséges egy oxigénatommag négy egyforma részre történő szétszakításához? Legalább mekkora energiájú neutron képes szétszakítani egy – kezdetben álló – oxigénatommagot?

(*4 pont*) *Példatári feladat nyomán*

P. 5087. Elméletileg lehet-e szabad szemmel észrevenni egy 80 km átmérőjű krátert a Hold felszínén, ha a pupillánk átmérője 5 mm?

(*4 pont*) *Csillagászati versenyfeladat*

P. 5088. Egy ℓ hosszúságú fonálingát vízszintesen kitérítünk, majd elengedünk. Amikor a fonál eléri a függőleges helyzetét, egy szögbe ütközik, s innen kezdve már csak az alsó, r hosszúságú része lendül tovább.



Mekkora az r/ℓ arány, ha az ingatest, miután felfelé haladva letér valahol a körpályáról, szabadon mozogva pontosan a szögbe ütközik?

(*6 pont*) Közli: *Radnai Gyula,* Budapest

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Problems in Mathematics

New exercises for practice – competition K (see page 541): **K. 604.** Find five appropriate distinct positive integers such that the sums of every possible selection of numbers out of those five is different. Find a set of five such numbers in which the largest number is as small as possible. **K. 605.** Define a cup as a figure formed by three small cubes pairwise sharing an edge in common (see the *figure*). We are building rectangular blocks out of small unit cubes. *a*) How many cups are there in a $4 \times 4 \times 2$ block? *b*) How many cups are there in a $4 \times 4 \times 3$ block? **K. 606.** The length of sides AB, BC, CD and DE of a pentagon $ABCDE$ is unity, $\angle ABC$ and $\angle CDE$ are both 90°. Show that the plane can be tiled with such pentagons without gaps or overlaps. Show it for both convex and concave pentagons. **K. 607.** Regular hexagons are constructed out of congruent regular triangles as shown in the *figure.* The first one consists of six triangles, the second one consists of twenty-four. *a*) How many triangles does the sixth such hexagon consist of? *b*) We have 2017 such regular triangles. If we use them to construct the largest possible hexagon of this kind, how many triangles are left over? **K. 608.** *a*) Show that there are infinitely many integers whose squares end in three digits of 4. *b*) Is there a positive integer whose square ends in four digits of 4?

New exercises for practice – competition C (see page 542): **Exercises up to grade 10: C. 1511.** B and C are interior points of a line segment AD such that $AB = CD$. Prove that $PA + PD \geqslant PB + PC$ for any point P of the plane. **C. 1512.** We are making 50-gram cubes out of three different mixtures of red, white and green play dough. In the first type of cube, the proportion of the colours is $3:2:0$, in the second type it is $1:3:1$, and in the third type it is $0:1:4$. How many of each type should we make if we want to use 1 kg of each colour altogether? **Exercises for everyone: C. 1513.** Show that every perfect cube can be expressed as a difference of two perfect squares. **C. 1514.** A unit square is divided into four isosceles triangles by connecting an interior point to the vertices. Find the minimum and maximum values of the product of the areas of the four triangles. **C. 1515.** Find the real solutions of the equation where k is an odd positive integer: $\left(1-x+x^2\right)\left(1-x+x^2-\dots+x^{2k}\right)=\left(1-x+x^2-\dots+x^{k+1}\right)^2$. **Exercises upwards of grade 11: C. 1516.** From point $P(16,7)$ of the coordinate plane, a tangent is drawn to the circle of radius $r = 5\sqrt{3}$ centred at $O(4,-2)$. Let P' denote the orthogonal projection of the point of tangency onto the line segment OP. Find the coordinates of P'. **C. 1517.** The fields of a chessboard are coloured in three colours as shown in the *figure.* A knight is placed on a random field of the chessboard, and a random (but correct) move is made with that knight. What is the probability that the knight will arrive on a field that has the same colour as the starting field?

New exercises – competition B (see page 544): **B. 4990.** Let n denote a natural number greater than one. Let $d(n)$ denote the number of positive divisors of n, and let the sum of the divisors be $\sigma(n)$. Show that $\sigma(n) > d(n)\sqrt{n}$. (*3 points*) (Proposed by *Zs. Sárosdi,* Veresegyház) **B. 4991.** Arthur and Belle take turns in colouring the edges of a cube red. In each step, they colour an edge that is skew to the previously coloured edge. Artur starts. The player who cannot do an appropriate coluring will lose the game. Which player has a winning strategy? (*3 points*) **B. 4992.** To start, each of the numbers $1, 2, \dots, n$ is coloured either red or blue. In every move, three distinct numbers in arithmetic progression are selected, and the colour of each of the three numbers is changed. For what values of n is it true that however the numbers $1, 2, \dots, n$ are coloured initially, it is possible to turn all of them red by applying an appropriate sequence of such steps? (*4 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 4993.** A square is drawn over each of legs BC and CA of a right-angled triangle ABC. Let D and E denote the vertices of the squares that lie opposite to C. Show that the circumscribed circle of triangle ABC passes through the midpoint of line segment DE. (*4 points*) **B. 4994.** Prove that if the cubic equation $x^3 + Ax^2 + Bx + C = 0$ has three distinct positive roots for some values of the real parameters A, B and C, then $A^2 + B^2 + 18C > 0$. (*4 points*) (*German problem*) **B. 4995.**

Let D, E and F, respectively, denote the midpoints of sides BC, CA and AB of a right-angled but not isosceles triangle ABC, let O be the centre of the circumscribed circle of the triangle, and let M be the orthocentre. The tangent drawn to the circumscribed circle at A intersects line EF at P, and the tangent drawn at point B intersects line FD at Q. Show that lines PQ and OM are perpendicular. (*5 points*) **B. 4996.** Given a line segment and one point that divides it 1 : 2, construct the other point dividing the line segment 1 : 2 by only using a straight edge. (*6 points*) **B. 4997.** Consider the following sequence $p_n(x)$ of polynomials with integer coefficients: let $p_0(x) = 0$, $p_1(x) = 1$, and for all $n \geqslant 2$ let $p_n(x) = p_{n-1}(x) + x \cdot p_{n-2}(x)$. Prove that if a polynomial $f(x)$ divides the polynomials $p_n(x)$ and $p_m(x)$ for some positive integers n, m then it also divides the polynomial $p_{(m,n)}(x)$. ((n, m) denotes the greatest common divisor of n and m. A polynomial $P(x)$ divides a polynomial $Q(x)$ if there exists a polynomial $R(x)$ of real coefficients for which $Q(x) = P(x)R(x)$.) (*6 points*)

New problems – competition A (see page 545): **A. 737.** 100 points are given in space such that no four of them lie in the same plane. Consider those convex polyhedra with five vertices that have all vertices from the given set. Prove that the number of such polyhedra is even. (Proposed by: *Gyula Károlyi,* Budajenő) **A. 738.** Prove that if $p(x)$ and $q(x)$ are monic real polynomials such that $p(x)q(x) = p(x^2 - 2)$ then $q(x) = p(-x)$. **A. 739.** Let $a_1, a_2, \dots$ be a sequence of real numbers from the interval $[0, 1]$. Prove that there is a sequence $1 \leqslant n_1 < n_2 < \dots$ of positive integers such that $A = \lim_{\substack{i,j\to\infty \\ i\neq j}} a_{n_i+n_j}$ exists, i.e., for every real number $\varepsilon > 0$ there is a N_ε such that $\left|a_{n_i+n_j} - A\right| < \varepsilon$ is satisfied for any pair of distinct indices $i, j > N_\varepsilon$. (*CIIM 10,* Colombia)

Problems in Physics

(see page 570)

M. 382. One end of a thin, flexible and not stretchable thread is attached to the topmost point on the rim of a cylinder of radius R. The cylinder is fixed and it has a horizontal axis. A small object is attached to the other end of the thread. In the equilibrium position the vertical part of the thread has a length of $L = 3R$. The object is displaced, as shown in the *figure,* and then it is released. The period of the motion – for a relatively large initial displacement – of the object depends on the "amplitude" A. Measure for several different values of A by what percent the period of this pendulum $T(A)$ differs from the period $T_0 = 2\pi\sqrt{L/g}$ of a simple pendulum of length L?

G. 653. One morning two locomotives start from the engine shed and travel in the same direction. The first one is a diesel engine and has a speed of 90 km/h, whilst the other one is an electric engine, which started 1.5 minutes after the diesel engine, and has a speed of 20 m/s. 10 minutes after the diesel engine started it meets a fast train coming along the neighbouring track in the opposite direction. What is the speed of the fast train if it meets with the electric engine 1.5 minutes after it met with the diesel one? **G. 654.** The height of the thermal water of temperature 30 °C in a spa pool of base 12 m × 20 m is 75 cm. The pool was then filled with 50 °C water up to the height of 1 m. Due to heat loss the temperature of the mixture is 2 °C less than it would be without the loss. What amount of heat was lost during mixing the water? **G. 655.** A solid brick of mass 27 kg is placed onto a horizontal tabletop. If it is placed onto one of its face then the pressure on the tabletop is 4500 Pa. When another face is in contact with the table then the pressure is 7200 Pa, and facing down to its third side the pressure on the tabletop 2700 Pa. What is the density of the brick? **G. 656.** An old hair dryer has two switches. If the first switch