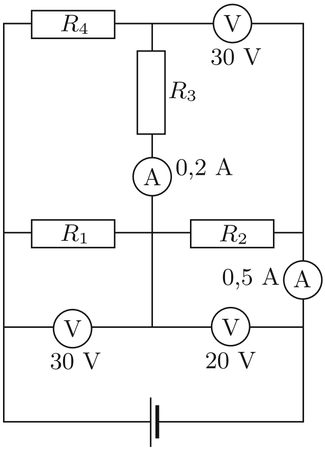
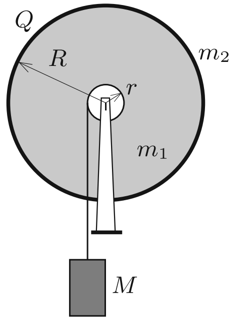
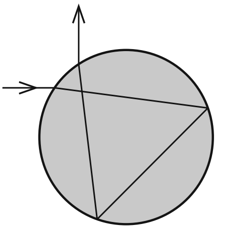


P. 5063.



P. 5064.



P. 5065.

P. 5065. Egy gömb alakú vízcseppre érkező fénysugár az *ábrán* látható módon, két belső visszaverődés után a bejövő sugárra merőleges irányban lép ki a vízcseppből. Mekkora a beesési szög? (A víz törésmutatója $n=\frac{4}{3}$.)

(*5 pont*) Közli: *Cserti József,* Budapest

P. 5066. Egy átlátszó közegben z irányban változik az optikai törésmutató. Erre merőlegesen, az x tengely irányában vékony fénysugarat indítunk, amely a közegben a pozitív z irányba eltérülve parabolaív mentén halad. A törésmutató értéke $z=0$-nál n_0, míg $z=h$-nál $\sqrt{2}\,n_0$. Hogyan függ a törésmutató z-től?

(*6 pont*) Közli: *Vigh Máté,* Budapest

Beküldési határidő: 2018. november 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

❄

MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 68. No. 7. October 2018)

Problems in Mathematics

New exercises for practice – competition K (see page 416): **K. 594.** Three two-digit prime numbers are formed by using one of the digits 2, 3, 5, 6, 7 twice and every other digit once. What is the sum of the three numbers? **K. 595.** QUARTO is a strategy board game (1991) for two players, invented by the Swiss mathematician Blaiseb Müller. The game includes a set of 16 pieces, each different from all others in some way. The pieces can be divided into two sets of eight by each of four different attributes: – tall or short; – black or white; – round or square; – hollow or solid at the top. In how many different ways is it possible to select two pieces that agree in exactly two or three attributes? **K. 596.** Let P, Q and R denote points on sides AB, BC and CA of a triangle ABC, respectively, such that $AP = AR$, $BP = BQ$ and $CQ = CR$ should hold. How many different sets of such points P, Q, R may exist for a given triangle ABC? **K. 597.** The midpoints P, Q, R and S of the sides of a square $ABCD$ are connected to the vertices as shown in the *figure.* Determine the ratio of the area of quadrilateral $BVDT$ to that of the square $ABCD$. **K. 598.** On a digital clock display, digits consist of small illuminated line segments as shown in the *figure.* The energy consumption of the clock depends on the number of small line segments switched on or off as time elapses. For example, when a 3 changes to a 4, two line segments are switched off and one is switched on, which means 3 switch operations. During a full cycle of $0, 1, 2, \dots, 9, 0$, this adds up to a total of 30 switch operations. If the same digit symbols were used to designate the numbers 0 to 9 in some different order, the number of switch operations could be reduced. Find the minimum number of switch operations that could be achieved in a full cycle, and give an example for a possible order of digits. (Proposed by *Zs. Ruttkai,* the Netherlands)

New exercises for practice – competition C (see page 417): **Exercises up to grade 10: C. 1497.** Solve the following simultaneous equations: $xy = z$, $xz = y$, $yz = x$. **C. 1498.** What is the maximum possible length of the shadow of a 2-metre-tall man on the Earth if the Earth is considered a sphere of radius 6370 km, illuminated by parallel light rays from the Sun? **Exercises for everyone: C. 1499.** Find the largest positive integer n for which there exists an appropriate order of the numbers $1, 2, \dots, n$ such that the large number obtained by writing all the numbers together in a row has the following property: for any pair a, b of successive digits, at least one of the two-digit numbers $\overline{ab}$, $\overline{ba}$ is a prime. **C. 1500.** On a line segment AB, the points X and Y are marked, and the squares $AXPQ$, $XBRS$, $BYWV$ and $YAUT$ are drawn, with their vertices labelled in counterclockwise order. The centres of the squares are denoted by K, L, M and N, respectively. Prove that the line segments KM and LN are perpendicular and equal in length. (*German competition problem*) **C. 1501.** Find the longest arithmetic sequence of distinct prime numbers less than 200. **Exercises upwards of grade 11: C. 1502.** In each *figure,* there are six circles of equal radius drawn in a unit square. In which arrangement do the circles have a larger radius? (*German competition problem*) **C. 1503.** In a triangle, the squares of the sides a, b, c, in this order, form an arithmetic sequence. Show that the measure of the angle opposite to side b is at most 60°.

New exercises – competition B (see page 418): **B. 4974.** At least how many numbers should be selected out of $1, 2, \dots, 10$ so that we can be certain that every such selection will contain a set of numbers whose sum is divisible by 11? (*3 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 4975.** Given a point P and four pairwise different lines $e \parallel f$ and $g \parallel h$, construct a line through P that intersects the lines e, f, g, h, respectively at points E, F, G, H such that $EF = GH$. (*3 points*) **B. 4976.** Let $A = \{-4; -3; -2; -1; 0; 1; 2; 3; 4\}$. First and Second take turns in selecting a number (not selected before) out of the elements of set A. The player first collecting three numbers that add up to zero wins the game. Is there a player who has a winning strategy? (*4 points*) (Proposed by *Á. Bán-Szabó,* Budapest) **B. 4977.** Prove that the orthocentre of the triangle formed by the points of tangency of the incircle on the sides of a right-angled triangle lies on the altitude drawn to the hypotenuse. (*4 points*) (*Kvant*) **B. 4978.** Let $n \geqslant 3$ be an integer and let α denote an arbitrary real number. Prove that $\sum_{k=0}^{n-1} \cos^2\left(\alpha + \frac{2k\pi}{n}\right) = \frac{n}{2}$. (*5 points*) **B. 4979.** In an acute angled triangle ABC, D and E are interior points of the sides AB and AC, respectively. The line segments BE and CD meet at F. Prove that if $BC^2 = BD \cdot BA + CE \cdot CA$ then the quadrilateral $ADFE$ is cyclic. (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 4980.** Let $n>3$ be a positive integer, and let $a_1, a_2, \dots, a_n$ be positive real numbers. Prove that $1 < \frac{a_1}{a_n+a_1+a_2} + \frac{a_2}{a_1+a_2+a_3} + \dots + \frac{a_n}{a_{n-1}+a_n+a_1} < \left[\frac{n}{2}\right]$ where the left-hand side of the inequality cannot be replaced by a larger number, and the right-hand side cannot be replaced by a smaller number. ($[x]$ denotes the greatest integer not greater than the number x.) (*6 points*) **B. 4981.** The area of the orthogonal projection of a unit cube onto the plane xy is A, and the length of its orthogonal projection onto the z-axis is a. Prove that $A=a$. (*6 points*) (Proposed by *P. Erben,* Budapest)

New problems – competition A (see page 420): **A. 734.** Let $G=(V,E)$ be a tree graph with n vertices, and let P be a set of n points in the plane with no three points collinear. Is it true that for any choice of the graph G and set P, we can embed G in P, i.e., we can find a bijection $f: V \to P$ such that when we draw the line segment $\left[f(x), f(y)\right]$ for all $(x,y) \in E$, no two such segments intersect each other? (Proposed by *Benedek Váli,* Szeged) **A. 735.** Does there exist an infinite sequence $a_1, a_2, \dots$ of real numbers which is bounded, not periodic, and satisfies the recursion $a_{n+1} = a_{n-1}a_n + 1$? **A. 736.** Circle ω lies in the interior of circle Ω, on which a point X moves. The tangents from X to ω intersect Ω for the second time at points $A \neq X$ and $B \neq X$. Prove that the lines AB are either all tangent to a fixed circle, or they all pass through a point.

Problems in Physics

(see page 442)

M. 380. Measure the rotational inertia about the symmetry axis of a hard boiled egg.

G. 645. In a NASA' vacuum chamber it was filmed that both a hammer and a feather fall towards the Earth at the same acceleration of $g = 9.81$ m/s^2, and if they are released at the same instant then they hit the ground at the same time. What will the acceleration of the feather and the hammer be if the film is played at twice the speed of the recording? **G. 646.** In a chemistry laboratory chemicals are stored in alike bottles. One is full of glycerine, and the other is full of ether. The mass of the bottle filled with glycerine is 2290 grams, and that of the other bottle with ether in it has a mass of 1471 grams. What is the mass of an empty bottle? **G. 647.** In two – seemingly alike – electric kettles the heating wire is bent into the shape of a regular hexagon. In one of the kettles the heating element was connected as shown in *figure a*), whilst in the other the heating element was connected as shown in *figure b*). In which kettle will the water start to boil sooner? **G. 648.** A small bug starts crawling from the vertex P of a wooden cube of edges 10 cm. At least how long does it take for the bug to reach the furthest vertex Q of the cube, if the speed of the bug is 1 cm/s? In how many different paths can the bug move in order to reach Q in the shortest time?

P. 5056. A spring with negligibly small mass and of spring constant 40 N/m is standing on the tabletop in a vertical position, and on its top there is a sheet, which also has negligible mass. A 0.2 kg small object is dropped to the sheet from a height of 0.4 m, measured from the level of the sheet. For how long will the small object be on the sheet if it does not stick to it? **P. 5057.** A small object of mass $m = 0.5$ kg and another of mass $3m$ are placed to an inclined plane of angle of elevation of $\alpha = 30°$. The two small objects are attached with a negligible-mass rigid rod of length $d = 50$ cm. The top part of the slope is frictionless, whilst on the bottom part of the slope the coefficient of friction is $\mu = 0.2$. Initially the object of mass m was at a distance of $L = 40$ cm from the boundary of that region where there is friction, and at a distance of $s = 120$ cm from the