the radius of the largest escribed circle of a triangle, and let R denote the radius of the circumscribed circle. Prove that $r_a \geqslant \frac{3}{2}R$. (*5 points*) (A problem from *Paul Erdős* (1913–1996)) **B. 4964.** Is it true that if the functions $f, g : \mathbb{R} \to [0, 1]$ are periodic and the function $f + g$ is also periodic then they have a period in common? (*6 points*) **B. 4965.** For a vector $\mathbf{x} \neq \mathbf{0}$, let $\mathbf{e_x} = \frac{\mathbf{x}}{|\mathbf{x}|}$. A given plane $\mathcal{S}$ is parallel, but not identical to the plane of a given (non-degenerate) triangle ABC. Show that there exists a unique $P \in \mathcal{S}$, such that the vector $\mathbf{e}_{\overrightarrow{PA}} + \mathbf{e}_{\overrightarrow{PB}} + \mathbf{e}_{\overrightarrow{PC}}$ is perpendicular to $\mathcal{S}$. (*6 points*)

New problems – competition A (see page 289): **A. 725.** Let $\mathbb{R}^+$ denote the set of positive real numbers. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the following equation for all $x, y \in \mathbb{R}^+$: $f\big(xy + f(y)^2\big) = f(x)f(y) + yf(y)$. (Proposed by: *Ashwin Sah*, Cambridge, Massachusetts, USA) **A. 726.** In triangle ABC with incenter I, line AI intersects the circumcircle of ABC at $S \neq A$. Let the reflection of I with respect to BC be J, and suppose that line SJ intersects the circumcircle of ABC for the second time at point $P \neq S$. Show that $AI = PI$. (Proposed by: *József Mészáros*, Galanta, Slovakia) **A. 727.** For any finite sequence $(x_1, \ldots, x_n)$, denote by $N(x_1, \ldots, x_n)$ the number of ordered index pairs (i, j) for which $1 \leqslant i < j \leqslant n$ and $x_i = x_j$. Let p be an odd prime, $1 \leqslant n < p$, and let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be arbitrary residue classes modulo p. Prove that there exists a permutation π of the indices $1, 2, \ldots, n$ for which $N\big(a_1 + b_{\pi(1)}, a_2 + b_{\pi(2)}, \ldots, a_n + b_{\pi(n)}\big) \leqslant \min\big(N(a_1, a_2, \ldots, a_n), N(b_1, b_2, \ldots, b_n)\big)$.

Problems in Physics

M. 378. Measure by what percent does a mosquito net (or any similar material) decrease the transparency of a window.

G. 637. Two balls are started at the same initial speed, each rolls along a horizontal plane first. During the motions both balls roll down along a slope, and then they both roll up to the initial level of their motion, and then they got to the end of the paths. The lengths of both paths are the same, the depths of the paths are also the same. Friction is negligible in both cases. Which ball reaches the end of the path first? **G. 638.** A 2-ton vehicle without operating its engine would move down a slope of 5% elevation at a constant speed of 36 km/h. What would the useful power of its engine be if it goes up along the same slope at the same speed? **G. 639.** It was observed that the flames of a burning candle has a spherical shape in a spaceship revolving around the Earth. Explain this observation. **G. 640.** If the Earth was a uniform density sphere of radius R, which of the *graphs* shown below would be the correct sketch of the gravitational force as a function of the distance measured from the centre of the Earth?

P. 5034. How long did the object, projected horizontally at an initial speed of v_0, fall while it reached a position which was at a distance of s from the position of the projection? (Neglect air resistance.) *Data:* $v_0 = 5$ m/s, $s = 20$ m. **P. 5035.** In winter a favourite type of food for titmice is a fat ball consisting of fat and different seeds. These balls are suspended by means of a piece of thread and hung to a branch of a tree. Even two tits can feed themselves from the same ball at the same time. Once there were two tits on the same ball of mass 90 g, when suddenly they got frightened and flew off at the same moment, with the same initial speed, in perpendicular directions, such that both tits initial velocity made an angle of $35°$ degree with the horizontal. The fat ball began to swing with a period of 1.4 s, the angular displacement of the thread (with respect to the vertical) was $10°$. The mass of each titmouse is 18 g. What was the initial speed

of the tits? **P. 5036.** The smallest distance between the Sun and a comet revolving around it is 0.5 AU and the greatest one is 31.5 AU. *a)* What is the period of the comet? *b)* What is the area which is swept by the line segment drawn from the Sun to the comet in one year? (Consider the Sun to be at rest.) **P. 5037.** A 50 cm long, 100 g mass chain of small links is hung such that its lower end is just above a scale. Suddenly the chain is released. Determine and sketch the reading on the scale as a function of the distance of the top of the chain and the scale; and as a function of the time elapsed from the release of the chain. **P. 5038.** Two short but strong boys are standing next to each other. One of them, called Andrew, throws a snowball at an initial speed of $v_0 = 10$ m/s at an angle of $\alpha = 30°$ with respect to the horizontal. His friend Bernard throws another ball at some velocity after the reaction time of $t_0 = 0.5$ s has elapsed. Bernard's goal is to make the two snowballs collide whilst they are in the air. What should the least speed of the second ball be in order that Bernard has a chance to hit Andrew's ball? (The boys are standing in a level field, their shoulders are at a height of $h = 1$ m. For the sake of simplicity do not consider air drag.) **P. 5039.** The lengths of two light, rigid rods are ℓ_1 and ℓ_2. To one end of each a small object is attached, one having a mass of m_1 and the other m_2. The other ends of the rods are attached to each other rigidly such that the angle between the rods is α. The system is pivoted at the attachment of the rods and can swing freely about a horizontal axis, in a plane determined by the rods. What is the period of the motion of the system when it is displaced a bit from its equilibrium position? **P. 5040.** Students are writing a test in a room of base area 50 m^2 and of height 3 m. The door of the room is opened, the temperature is 24 °C, and the pressure is 10^5 Pa. Estimate the following quantities: *a)* What is the mass of the air in the room? *b)* What is the internal energy of the air in the room? *c)* By what amount would the internal energy of the air in the room change, if the temperature increases by 2 °C? **P. 5041.** There is a pair of fixed, parallel rails of negligible resistance in a horizontal plane at a distance of $L = 10$ cm. The rails are connected at one of their ends to the terminals of a voltage supply of constant voltage of $U_0 = 0.3$ V, as shown in the *figure.* The system is in downward magnetic field of magnetic induction of $B = 1$ T. A metal rod of resistance $R = 0.2$ Ω is placed perpendicularly to the rails. The rod can move frictionlessly on the rails. What is the magnitude and the direction of the force which is to be exerted on the rod in order that it moves at a constant speed of v in the direction shown in the figure if *a)* $v = 1$ m/s; *b)* $v = 5$ m/s? *c)* What is the dissipated power of the battery in both cases? **P. 5042.** A beam of magenta light, which is combined from red light of wavelength 652 nm, and blue light of wavelength 489 nm, is perpendicularly incident on a traditional diffraction grating. The distance between the two closest magenta coloured spots on a screen at a distance of 2 m is 20 cm. What is the slit spacing of the diffraction grating? **P. 5043.** A deuterium of kinetic energy $1.6 \cdot 10^{-13}$ J collides with a stationary tritium. The following nuclear reaction occurs: $^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$. The angle between velocity of the emitted neutron and the velocity of the deuterium is 60°. *a)* How much energy is released? *b)* What is the kinetic energy of the α-particle and the neutron, after the collision? *c)* What is the angle between the velocities of the α-particle and the deuterium? **P. 5044.** Alex and Bob are twins. Their fate changes on their 20th birthday: Alex stays on the Earth, but Bob goes to a longer space-expedition. The spaceship is travelling away the Earth at a constant speed. A year later Alex takes a photo of his birthday cake, and sends it to Bob by means of radio signals, who receives it on his 22nd birthday in the spaceship. *a)* At what speed does the spaceship travel away from the Earth? *b)* According to Alex how far is the spaceship from the Earth when Bob receives the photo? *c)* Bob also takes a photo of his 22nd birthday, and immediately sends it to Alex. How old is Alex when he receives the photo?