rooms are there? (*4 points*) (Proposed by *A. Faragó* and *T. Káspári,* Paks) **B. 4943.** There is an ant at each corner of a given face of a rectangular brick. Each ant wants to get to the opposite vertex of the cuboid, that is, to the other endpoint of the space diagonal drawn from his vertex of the cuboid. Is it possible for the ants to crawl to the opposite vertices along the surface of the brick, so that they follow the shortest possible paths and their paths do not intersect? (*4 points*) (Proposed by *M. E. Gáspár,* Budapest) **B. 4944.** Let t denote the area of (some) triangle of maximum area inscribed in a convex plane figure $\mathcal{S}$, and let T denote the area of (some) triangle of minimum area circumscribed about $\mathcal{S}$. What is the maximum of the ratio $\frac{T}{t}$? (*5 points*) **B. 4945.** Find all positive integers n for which $1\cdot 2^0+2\cdot 2^1+3\cdot 2^2+\ldots+n\cdot 2^{n-1}$ is a perfect square. (*5 points*) (Based on the idea of *L. Németh,* Fonyód) **B. 4946.** Let $f(x)$ be a polynomial of real coefficients such that $f(k)$ is an integer for every positive integer k that ends in 5 or 8 in decimal notation. *a*) Prove that $f(0)$ is an integer. *b*) Give an example of a polynomial $f(x)$ that meets the above conditions, but $f(1)$ is not an integer. (*6 points*) **B. 4947.** Prove that there is exactly one way of dissecting a cube into five tetrahedra. (Two dissections are not considered different if the resulting pieces are congruent.) (*6 points*)

New problems – competition A (see page 163): **A. 719.** Let ABC be a scalene triangle with circumcenter O and incenter I. The A-excircle, B-excircle, and C-excircle of triangle ABC touch BC, CA, and AB at points A_1, B_1, and C_1, respectively. Let P be the orthocenter of AB_1C_1 and H be the orthocenter of ABC. Show that if M is the midpoint of PA_1, then lines HM and OI are parallel. (Proposed by: *Michael Ren,* Andover, Massachusetts, USA) **A. 720.** We call a positive integer *lively* if it has a prime divisor greater than $10^{10^{100}}$. Prove that if S is an infinite set of lively positive integers, then it has an infinite subset T with the property that the sum of the elements in any finite nonempty subset of T is a lively number. **A. 721.** Let $n\geqslant 2$ be a positive integer, and suppose $a_1,a_2,\ldots,a_n$ are positive real numbers whose sum is 1 and whose squares add up to S. Prove that if $b_i=\frac{a_i^2}{S}$ $(i=1,\ldots,n)$, then for every $r>0$, we have

$$\sum_{i=1}^{n}\frac{a_i}{(1-a_i)^r}\leqslant\sum_{i=1}^{n}\frac{b_i}{(1-b_i)^r}.$$

Problems in Physics

(see page 186)

M. 376. A half-litre bottle is filled with water and is made swing about a horizontal axis, which is perpendicular to the bottle's symmetry axis, and goes through the cap of the bottle. Measure the period of the pendulum for different initial angular displacements. Will the result change if the water is frozen in the bottle?

G. 629. Once, when Nasreddin Hodja shouldered his heavy pack and got on his donkey with the pack, he was asked why he did not put his pack to the donkey. He answered: „Because that would be cruelty to animals, I am heavy enough for this poor little thing". *a*) Why is this answer wrong? *b*) Draw the forces acted upon the objects mentioned in the story. **G. 630.** Why does the surface of the water in a rotating container have concave shape? **G. 631.** A current of 2 A is flowing through a 30 g copper wire across which there is a voltage of 1.2 V. What should the voltage across that copper wire be which is also 30 g, but twice as long as the other one and the same 2 A current flows through it? **G. 632.** A plane, which is flying at a speed of 900 km/h, uses 4 litres of fuel (kerosene) in each second. What distance is covered in each minute by that car which has a fuel consumption of 6.4 litres of petrol per 100 km and which needs the same amount of petrol in 5 hours as the amount of kerosene consumed by the plane while it covers a distance of one kilometre?

P. 5012. Team Hungary won men's 5000 m short track speed skating relay gold at PyeongChang Winter Olympic Games and claimed champion in a time of 6:31.971 and set a new Olympic record. The short track of total length 111.12 m consists of two straight and two semi-circular segments with radius 8.5 m. Estimate the skaters' angle of lean in the turns of the track. **P. 5013.** The total mass of the trolley, which has small light wheels, and the ring of radius $R=1$ m on the trolley is m (see the *figure*). A small point-like object of mass m is placed to the bottom of the ring. The small object is given an initial speed of v_0. What is the value of v_0 if the trolley just rises from the ground when the object reaches the topmost point of the ring? Friction is negligible everywhere. **P. 5014.** At what speed should a projectile be projected on the Moon, in order that the height to which it rises is p percent of the radius of the Moon? Let the values of p be the following: $p=1$, 10 and 100. (Give your answers to 2 significant figures.) **P. 5015.** The star called 55 Cancri has the same diameter and mass as the Sun has. Its innermost planet Janssen has an orbital period of 17.76 hours. Determine the average distance between the star and the planet in astronomical units, which is the average distance between the Sun and the Earth. **P. 5016.** There is a uniform-density rod on the horizontal tabletop. We would like to bring the rod slowly to a vertical position with a force which is exerted at one end of the rod and which is perpendicular to the rod during the whole process. What is the least value of the coefficient of static friction, if the rod does not slip? **P. 5017.** A heating element is built into a boiler of 10 liters. The element has such a small power that it is unable to heat the water to its boiling point. After heating the water totally, during the first minute after ceasing the heating the temperature of the water decreases by 1 °C. What is the power rating of the heating element if the water equivalent of the calorimeter is 3 kg? **P. 5018.** If fuel is burnt in a heat engine and a heat pump is operated with the heat engine then more heat can be transferred into the flat than in the case when the fuel is burnt in a stove. Let the flat be the low temperature heat reservoir of the heat engine and the high temperature heat reservoir of the heat pump. The cold temperature heat reservoir of the heat pump can be the air of the street. Suppose that the efficiency of the heat engine is η_1, and that the efficiency of the heat pump if it was operated as a heat engine would be η_2. Calculate the factor by which the heat transferred to the flat by means of the heat engine and the pump is greater than the heat transferred to the flat by burning the fuel in the stove. **P. 5019.** An electron of energy 1.5 eV is moving in a uniform vertical magnetic field of induction $2\cdot 10^{-3}$ T, such that the angle between its velocity vector and the horizontal is 30°. How many times does it cross the same induction line while it descends 20 cm? **P. 5020.** A circular hole on a screen is illuminated by a coherent laser beam perpendicular to the screen. Behind the screen and perpendicular to the optical axis a CCD-detector sheet was placed. By what percent does the illumination of the pixel on the optical axis (the intensity of the incident light beam) decrease if one-sixth of the hole is covered by an opaque sheet having a circular sector shape? **P. 5021.** At most how much energy can be gained by an – initially stationary – electron if it collides with another particle of energy 1 MeV, if the particle is a *a*) proton; *b*) electron; *c*) positron? **P. 5022.** The lengths of two threads are L and $2L$. At the ends of the threads there are point-like objects of mass m. The objects have the same Q charge. What is the angle between the two threads which are fixed at the same point in equilibrium? *Data:* $L=20$ cm, $m=1$ g, $Q=2.8\cdot 10^{-7}$ C.