(*5 pont*) *Példatári feladat nyomán*

P. 5021. Legfeljebb mekkora energiára tehet szert egy – kezdetben állónak tekinthető – elektron, ha egy 1 MeV mozgási energiájú másik részecskével ütközik, amennyiben ez a részecske

a) proton;

b) elektron;

c) pozitron?

(*4 pont*) Közli: *Fröhlich Georgina,* Budapest

P. 5022. Két fonál közül az egyik L, a másik $2L$ hosszúságú. A fonalak végein azonos, m tömegű, pontszerűnek tekinthető testek vannak. A testeknek azonos, Q töltése van. Egyensúly esetén mekkora szöget zárnak be a közös pontban rögzített fonalak?

Adatok: $L = 20$ cm, $m = 1$ g, $Q = 2{,}8 \cdot 10^{-7}$ C.

(*6 pont*) Közli: *Zsigri Ferenc,* Budapest

❄

Beküldési határidő: 2018. április 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

❄

MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 68. No. 3. March 2018)

Problems in Mathematics

New exercises for practice – competition K (see page 160): **K. 583.** An integer is said to be a *prime-rose* if its first digit is a prime, the sum of the first two digits is also a prime, the sum of the first three digits is also a prime, and so on. Find the largest prime-rose number in which all digits are different. **K. 584.** Santa Claus is very strong, but he can only carry a maximum of 100 kg of presents in his sack. In a large apartment building, he was to deliver three different kinds of presents: A, B and C. The mass of each type of present is a whole number of kilograms. He can carry eight A and eight B at the same time, but in that case he cannot take any further piece (neither A, nor B or C) in that round. Similarly, he cannot take anything further if he carries ten A, four B and four C. How much may each of the presents A, B and C weigh in kilograms? **K. 585.** Andrew wrote three (not necessarily different) positive integers on a blackboard, each of them smaller than 2018. Then he erased these numbers (A, B and C), and replaced them with $\frac{A+B}{2}$, $\frac{B+C}{2}$, $\frac{A+C}{2}$. He repeated this procedure 11 times altogether. As a result, one of the three numbers on the board is 100. What are the other two numbers? **K. 586.** The distances of an interior point of a regular hexagon from three consecutive vertices are 4, 4 and 8 units. How long are the sides of the hexagon? **K. 587.** How many of the numbers 2014, 2015, 2016 and 2017 can be expressed as a sum of squares of six not necessarily different odd numbers? **K. 588.** Let $A > B$ be four-digit numbers such that B is obtained by writing the digits of A in reverse order. What are the smallest and largest possible values of $A - B$?

New exercises for practice – competition C (see page 161): **Exercises up to grade 10: C. 1469.** The foot of the altitude drawn from vertex C of a triangle ABC is T, an interior point of side AB. The angle bisector drawn from C intersects AB at R. Given that $AB = 10$, $AT = 3$ and $AR = 4$, find the lengths of the sides of the triangle. **C. 1470.** What is the radius of two touching congruent spheres centred at the centres of two adjacent faces of a unit cube? **Exercises for everyone: C. 1471.** Prove that every power of two greater than four can be expressed as the difference of two odd square numbers. For example, $32 = 81 - 49$. **C. 1472.** A certain game involves collecting cards with various things on them. Each card has exactly two of the following 9 things: colours (red, white, or green), elements (air, earth, fire, or water) and animals (rabbit or sheep). A card shows at most one of each category. In how many different ways is it possible to select four cards such that there are eight different things on them, provided that the game contains all possible combinations? **C. 1473.** The number abc is expressed in base $2a$ notation. What is the base if $c - b = b - a = 1$, and the value of abc equals $29a^2 + 9a + 9$ in decimal notation? **Exercises upwards of grade 11: C. 1474.** Let P, Q and R denote the feet of the altitudes of the acute-angled triangle ABC. Given that $BP : PA = 1 : 2$ and $AQ : QC = 3 : 1$, find the proportion of the pieces formed by R on side BC. **C. 1475.** What is the largest possible area of the lateral surface of a cylinder inscribed in a unit sphere?

New exercises – competition B (see page 162): **B. 4939.** Show that a convex 2018-sided polygon cannot be dissected into triangles in which the angles in degrees are all integers. (*3 points*) **B. 4940.** What may be the value of the sum $x + y + z$ if $x^4 + 4y^4 + 16z^4 + 64 = 32xyz$? (*3 points*) **B. 4941.** The centre O of the circumscribed circle of an acute-angled triangle ABC is reflected in the feet of the altitudes. Prove that the circle formed by the three reflections has the same radius as the circumscribed circle of the triangle. (*4 points*) **B. 4942.** The one hundred mathematicians participating in an international combinatorial conference were all housed in the same hotel. The receptionist was originally planning to place them in the order of their arrival in the rooms numbered 1 to 100. However, he forgot to give that instruction to the guest arriving first, who has thus chosen a room at random. So the receptionist instructed all the other guests to take the room with their number in the order of arrivals, or, if that room has already been taken, to select any other room they like. How many possible arrangements of the guests in the rooms are there? (*4 points*) (Proposed by *A. Faragó* and *T. Káspári,* Paks) **B. 4943.** There is an ant at each corner of a given face of a rectangular brick. Each ant wants to get to the opposite vertex of the cuboid, that is, to the other endpoint of the space diagonal drawn from his vertex of the cuboid. Is it possible for the ants to crawl to the opposite vertices along the surface of the brick, so that they follow the shortest possible paths and their paths do not intersect? (*4 points*) (Proposed by *M. E. Gáspár,* Budapest) **B. 4944.** Let t denote the area of (some) triangle of maximum area inscribed in a convex plane figure $\mathcal{S}$, and let T denote the area of (some) triangle of minimum area circumscribed about $\mathcal{S}$. What is the maximum of the ratio $\frac{T}{t}$? (*5 points*) **B. 4945.** Find all positive integers n for which $1\cdot 2^0+2\cdot 2^1+3\cdot 2^2+\ldots+n\cdot 2^{n-1}$ is a perfect square. (*5 points*) (Based on the idea of *L. Németh,* Fonyód) **B. 4946.** Let $f(x)$ be a polynomial of real coefficients such that $f(k)$ is an integer for every positive integer k that ends in 5 or 8 in decimal notation. *a*) Prove that $f(0)$ is an integer. *b*) Give an example of a polynomial $f(x)$ that meets the above conditions, but $f(1)$ is not an integer. (*6 points*) **B. 4947.** Prove that there is exactly one way of dissecting a cube into five tetrahedra. (Two dissections are not considered different if the resulting pieces are congruent.) (*6 points*)

New problems – competition A (see page 163): **A. 719.** Let ABC be a scalene triangle with circumcenter O and incenter I. The A-excircle, B-excircle, and C-excircle of triangle ABC touch BC, CA, and AB at points A_1, B_1, and C_1, respectively. Let P be the orthocenter of AB_1C_1 and H be the orthocenter of ABC. Show that if M is the midpoint of PA_1, then lines HM and OI are parallel. (Proposed by: *Michael Ren,* Andover, Massachusetts, USA) **A. 720.** We call a positive integer *lively* if it has a prime divisor greater than $10^{10^{100}}$. Prove that if S is an infinite set of lively positive integers, then it has an infinite subset T with the property that the sum of the elements in any finite nonempty subset of T is a lively number. **A. 721.** Let $n\geqslant 2$ be a positive integer, and suppose $a_1,a_2,\ldots,a_n$ are positive real numbers whose sum is 1 and whose squares add up to S. Prove that if $b_i=\frac{a_i^2}{S}$ $(i=1,\ldots,n)$, then for every $r>0$, we have

$$\sum_{i=1}^{n}\frac{a_i}{(1-a_i)^r}\leqslant\sum_{i=1}^{n}\frac{b_i}{(1-b_i)^r}.$$

Problems in Physics

(see page 186)

M. 376. A half-litre bottle is filled with water and is made swing about a horizontal axis, which is perpendicular to the bottle's symmetry axis, and goes through the cap of the bottle. Measure the period of the pendulum for different initial angular displacements. Will the result change if the water is frozen in the bottle?

G. 629. Once, when Nasreddin Hodja shouldered his heavy pack and got on his donkey with the pack, he was asked why he did not put his pack to the donkey. He answered: „Because that would be cruelty to animals, I am heavy enough for this poor little thing". *a*) Why is this answer wrong? *b*) Draw the forces acted upon the objects mentioned in the story. **G. 630.** Why does the surface of the water in a rotating container have concave shape? **G. 631.** A current of 2 A is flowing through a 30 g copper wire across which there is a voltage of 1.2 V. What should the voltage across that copper wire be which is also 30 g, but twice as long as the other one and the same 2 A current flows through it? **G. 632.** A plane, which is flying at a speed of 900 km/h, uses 4 litres of fuel (kerosene) in each second. What distance is covered in each minute by that car which has